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I. *The Radiation and Convection from a Heated Wire in an Enclosure of Air.* By THOMAS BARRATT, *A.R.C.S., D.Sc.*

RECEIVED JUNE 22, 1915.

1. *Objects of Experiment.*

WHEN the temperature of a metallic wire or cylindrical rod placed in the open air or in an enclosure at constant temperature containing a gas is raised above that of the gas, it is found that the loss of heat H from the wire is very nearly proportional to its excess of temperature V above that of the gas surrounding it. Assuming that the loss of heat is entirely due to convection, Boussinesq * has obtained this result mathematically in the case of the convection of heat by a stream of liquid from a cylinder maintained at constant temperature. Experimental results by Compan † (in the case of spheres heated in air) and Kennelly ‡ and others (for metallic wires heated in air by an electric current) have verified this conclusion. The truth of Newton's law ($H \propto V$) has also been assumed, with justifiable results, in its application to various instruments of precision, such as hot-wire ammeters, and, quite recently, anemometers.§ In an arrangement of apparatus adopted by the present author ‖ for the measurement of thermal conductivity (*see* fiigure on next page), the law was found to be accurately true up to an excess of temperature of, at any rate, 12°C.

It seems to be generally admitted that Newton's law is applicable, with great accuracy, to that portion of the heat which is lost by convection, even when the temperature of the heated body is raised many degrees above that of its surroundings, but that the radiation loss by no means conforms to that law. Assuming for the moment that Stefan's "fourth power" law applies to the part of the heat, R lost by radiation from a heated metallic wire, we have

$$R=\sigma(\theta^4-\theta_0^4),$$

* Boussinesq, "Comp. Rend.," CXXXIII., p. 257, 1901. *See also* A. Russell, "Proc." Phys. Soc., XXII., p. 432, 1909; and L. V. King, Roy. Soc. "Phil. Trans.," 373, Nov. 12, 1914.

† P. Compan, "Ann. de Chim. et Phys.," XXVI., p. 488, 1902.

‡ Kennelly and Sanborn, Amer. Phil. Soc. "Proc.," 55-77, 1914.

§ J. T. Morris, "The Electrician," p. 1056, Oct. 4, 1912; "Engineering," Aug. 8, 1913.; L. V. King, *loc. cit.*

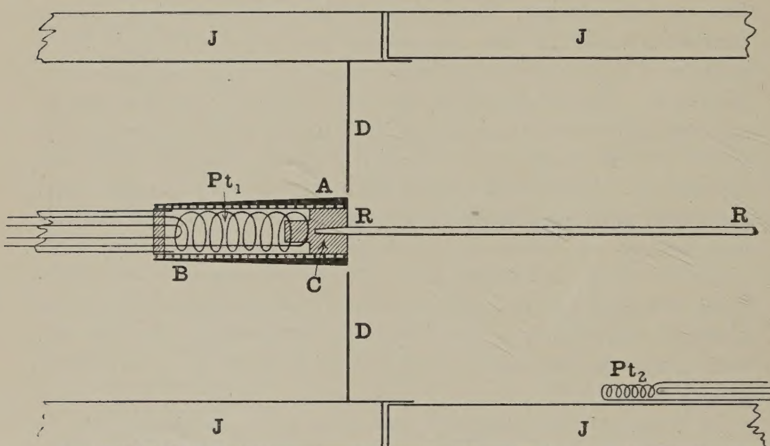
‖ T. Barratt, "Proc." Phys. Soc., XXVI., V., 346, Aug. 1914.

where σ is constant, and θ and θ_0 are the absolute temperatures of the hot body and its surroundings respectively. If we put $\theta=\theta_0+x$, we have

$$R=\sigma x 4\theta_0^3\{1+3x/(2\theta_0)+x^2/\theta_0^2+x^3/(4\theta_0)^3\}.$$

Hence, R is proportional to x provided that $3x/(2\theta_0)$ can be neglected compared to unity. (For example, the heat lost for 2°C. excess at air temperature is twice that lost for 1°C. excess to within 1 part in 200.)

If then the radiation loss is only a small percentage of the convection, we should expect Newton's law for the convective



RR=rod or wire; C=solid copper cylinder; AB=copper cylinder tapering in thickness from 2 mms. to zero; Pt_1 and Pt_2=platinum thermometers; DD=thin copper disc dividing enclosure into two compartments; JJJJ=water or steam jacket.

heat loss to apply for the total heat loss for small differences of temperature.

As far as one can find from recent literature on the subject, no experimental work has been attempted with the direct object of determining the relation between the amounts of radiation and convection from a body heated in an enclosure of gas. Kennelly * was uncertain of the correction for radiation from a hot wire, but finally assumed that a copper wire radiates 94 per cent. as much heat as a black body at the same tempera-

* A. E. Kennelly, " Trans." Amer. Inst. Elec. Eng., XXVII., I., 363, 1909.

ture. This correction then amounted to about 2 per cent. for the smallest wire, and about 8 per cent. of the total energy for the largest wire employed.

Langmuir, however, referring to the above Paper, remarks : "Hence it would have been better for Kennelly to neglect the small percentage due to radiation from copper, as the radiation is probably less than 1 per cent." (Irving Langmuir, Amer. Inst. Elec. Eng., June 25, 1912.) In a previous Paper Langmuir ("Phys. Rev.," XXXIV., p. 401, 1912) asserts that: "Radiation from small metallic wires is practically negligible compared to convection up to several hundred degrees." L. V. King (*loc. cit.*) says : "In the present work the heat lost by radiation plays a very subordinate part, and was not made the subject of special investigation."

It seemed desirable, therefore, to attempt to determine, by a direct method, the precise numerical relation between the radiation and the convection from a wire heated by an electric current in an enclosure of air maintained at constant temperature, especially as an arrangement of apparatus recently employed by the author for the measurement of thermal conductivity was admirably adapted for the purpose.

II. Experimental Procedure.

The wire or rod RR (*see* Figure) was supported in the middle of an enclosure of 5 cm. diameter, consisting of a brass water or steam jacket, JJJJ. One end of the wire, of slightly conical shape, was inserted in a hole bored in a solid copper cylinder, C, which fitted tightly into a hollow copper cylinder. Around the latter was wound 3 metres of pure platinum wire, gauge 30, single silk covered, and over this was another copper cylinder, AB. The end of the rod RR was raised to a temperature, V, above that of the enclosure by sending an electric current through the platinum coil. The temperatures of the hot end of the wire and of the enclosure were given by two platinum thermometers Pt_1 and Pt_2 respectively. After allowing adequate time (an hour or more) for temperature conditions to become steady, the current C through the platinum coil, and the E.M.F. E at its ends were measured by means of a carefully calibrated potentiometer. The specimen wire or rod was then removed, and the current adjusted so as to keep $Pt_1 - Pt_2$ precisely the same as when the wire was in position. If the current and E.M.F. be now denoted by C′ and E′, it is clear that the heat H given to the wire in order to maintain

A 2

the temperature of its extremity $V(=Pt_1-Pt_2)$ above that of the enclosure is given by $CE-C'E'$.

It has been shown in previous Papers (*loc. cit.*) that for a rod of given dimensions (perimeter p, cross-section q, length l) and thermal conductivity k, whose end is raised V^0 above the temperature of the enclosure,

$$h=\frac{H^2 \coth {}^2 al}{pqk\ V^2},$$

where

$$a=\sqrt{hp/qk}.$$

Hence,

$$h \propto H^2,$$

or the heat h lost per second from a square centimetre of surface for 1°C. excess is proportional to the *square* of the total heat loss, the latter being measurable.

The following experiments were therefore carried out :—

1. The amount of heat H ($=CE$) was determined that was necessary to raise the temperature of the hot end of a brass rod a given amount V (about 10°C.),

(*a*) With the surface of the rod in its usual state—*i.e.*, slightly soiled by its exposure to the air,

(*b*) With the surface of the rod lightly and evenly coated with a dead-black varnish (an aniline dye dissolved in amyl acetate),

(*c*) With the rod removed.

2. A " Leslie cube " of 20 cm. edge was constructed of brass, and one of its faces was evenly coated with the same dead-black varnish as was employed in experiment (1) (*b*). The cube having been filled with water at a temperature a few degrees above that of the room, the relative " emissive powers " of the " metallic " and " black " surfaces were determined by means of a thermopile in connection with a low resistance galvanometer. (It was considered preferable to use a thermopile in preference to (say) a bolometer, since the former is steadier, and in addition, measures the total emission of all wavelengths.)

Now,

$$h_1/h_2 \text{ or } H_1{}^2/H_2{}^2=\frac{CE-C'E'}{C_1E_1-C'E'},$$

where

C, E are the current and E.M.F. when the rod is " black."
$C_1\ E_1$ „ „ „ „ „ „ " bright."
$C'\ E'$ „ „ „ „ „ „ removed.

Let $$H_1^2=aH_2^2, \text{ i.e., } h_1=ah_2, \qquad (1)$$

as given by the first set of experiments.

Also let c=heat lost by convection in each case (assumed equal for both surface conditions of the rod).

Let r_1/r_2 be the measured ratio of amounts of heat lost by radiation from "black" and "bright" surfaces, as given by the second set of experiments, and suppose

$$r_1=br_2. \qquad (2)$$

Hence, from (1) $$c+r_1=a(c+r_2). \qquad (3)$$

From (2) and (3) $$\frac{r_2}{c}=\frac{a-1}{b-a}, \qquad (4)$$

and $$\frac{r_1}{c}=\frac{b(a-1)}{b-a}. \qquad (5)$$

Also $$\frac{r_2}{h_2}=\frac{r_2}{c+r_2}=\frac{a-1}{b-1}, \qquad (6)$$

and $$\frac{r_1}{h_1}=\frac{r_1}{c+r_1}=\frac{b(a-1)}{a(b-1)}. \qquad (7)$$

Equations (4) and (5) give at once the ratio of radiation to convection from the wire or rod when "bright" and "black" respectively, while (6) and (7) give the corresponding ratios of "radiation loss" to "total loss" of heat.

III. Experimental Results.

1. *Total Losses of Heat from Wire.*—Experiments were conducted with various wires, and employing different excesses of temperature. The rod (or wire) being put in position, and the current turned on and left until the steady state was reached, readings of C, E, Pt_1 and Pt_2 were obtained. The rod was then lightly varnished with the dead-black solution, without being moved from its position. After allowing sufficient time, the same measurements were again made. These operations were usually repeated two or three times to ensure consistent results. The rod was then removed, and C and E again determined, the excess of temperature, V (=Pt_1—Pt_2), being kept

constant throughout. The following observations were made in the case of a brass rod :—

Brass Rod : Diameter, 3 mm. ; length, 30 cm.

(*a*) *Jan.* 16, 1915 : Pt_1, 17·12°C. ; Pt_2, 12·18°C. ; "V," 4·94°C.

	Black.	Bright.	Rod out.	h_1/h_2.
E	4,687	4,637	3,588	1·109
C	297·6	294·5	227·8	

$$\frac{h_1}{h_2}=\left(\frac{H_1}{H_2}\right)^2=\left(\frac{4{,}687\times 297{\cdot}6-3{,}588\times 227{\cdot}8}{4{,}637\times 294{\cdot}5-3{,}588\times 227{\cdot}8}\right)^2=1{\cdot}109.$$

(*b*) *Jan.* 9, 1915 : Pt_1, 19·01°C. ; Pt_2, 11·21°C. ; "V," 7·80°C.

	Black.	Bright.	Rod out.	h_1/h_2.
E	6,175	6,119	4,834	1·113
C	388·0	383·25	300·8	

(*c*) *Jan.* 18, 1915 : Pt_1, 20·34°C. ; Pt_2, 10·05°C. ; "V," 10·29°C.

	Black.	Bright.	Rod out.	h_1/h_2.
E	7,027	6,847	5,287	1·118
C	444·0	439·6	335·0	

An experiment with a *silver wire* gave the results :—

(*d*) *Jan.* 27, 1915 : Diameter, 1 mm. ; length, 35 cm.
Pt_1, 23·29°C. ; Pt_2, 12·98°C. ; "V," 10·31°C.

	Black.	Bright.	Rod out.	h_1/h_2.
E	6,256	6,217	5,468	1·113
C	402·1	399·7	352·0	

In the case of a *copper wire*, the ratio h_1/h_2 was 1·109.

From the figures quoted it is evident that :—

(*a*) The increase in the emission of heat from a blackened rod or wire is about 11·5 per cent. ; in other words, the value of "a" in the equations derived above is 1·115.

(*b*) The effect of coating the rods with "dead-black" is about the same for all the metals examined, indicating that the amounts of heat lost by radiation from rods of different metals are the same, the rods being unblacked (*i.e.*, only slightly soiled by exposure to the air).

(*c*) The effect is, within the limits examined, independent of the diameter of the wire.

2. *Relative Losses of Heat by Radiation from "Metallic" and from "Dead Black" Surfaces.*—The "Leslie Cube" was filled with warm water, and the experiments conducted as already

described, as far away as possible from a fire or other source of heat. The thermopile employed consisted of 24 " elements " (Bi—Sb). A commutator was used in conjunction with the galvanometer, so that readings could be taken on both sides of the zero. Observations were made in the following order: " Black," " Bright," " Black," at equal intervals, and the average of the first and third compared with the second.

Radiation Experiments.

Dec. 29, 1914 : Distance of scale from galvanometer, 2 metres.			
First experiment: Temperature of " cube," 12°C. ; air temperature, 8°C. ; difference, 4°C.			
	Black surface.	Bright surface.	Ratio.
Deflection of galvanometer...	261 mm.	48·5 mm.	5·38 : 1
Second experiment: Temperature of " cube," 22°C. ; air temperature, 8°C. ; difference, 14°C.			
	Black surface.	Bright surface.	Ratio.
Deflection of galvanometer...	343 mm.	62·5 mm.	5·49 : 1
Third experiment: Temperature of " cube," 38°C. ; air temperature, 8°C. ; difference, 30°C.			
	Black surface.	Bright surface.	Ratio.
Deflection of galvanometer...	289 mm.	52 mm.	5·55 : 1
(The galvanometer was suitably shunted so as to give reasonable deflections in each case.)			

The results indicate that the relative amounts of radiation from " black " and " brass " surfaces are very nearly the same for all differences of temperature, the ratio increasing only slightly with increase of temperature excess. The approximate ratio is 5·5 : 1. That is, the value of " b " in equation (2) is 5·5.

Also " a " has been found $=1{\cdot}115$.

Hence, from equation (4)

$$\frac{r_2}{c}=\frac{1{\cdot}115-1}{5{\cdot}5-1{\cdot}115}=0{\cdot}0262,$$

and from (5),

$$\frac{r_1}{c}=0{\cdot}144.$$

That is, the radiation in the case of the " bare " metal is 2·62 per cent. of the convection, and for the " blackened " metal is 14·4 per cent.

Also from (6) and (7),

$$\frac{r_2}{h_2}=\frac{a-1}{b-1}=2{\cdot}5 \text{ per cent.}$$

$$\frac{r_1}{h_1}=\frac{b(a-1)}{a(b-1)}=12{\cdot}6 \text{ per cent.}$$

Or, *of a total loss of heat of* 100 *parts*, 2·5 *are due to radiation, and* 97·5 *to convection* (*for a " bare " wire*). *If the wire is coated with " dead-black," the corresponding percentages are* 12·6 *and* 87·4.

IV. Experiments at Higher Temperatures.

The value of " h," the total loss of heat per second from 1 sq. cm. of surface of the wire for 1°C. excess, is considerably greater at 100°C. than at ordinary air temperatures. For example, the author obtained the values, " h " at 17°C. =0·000533, and at 100°C.=0·000634, for a wire 1 mm. in diameter. Lees,* in the case of a rod 5·85 mm. in diameter, obtained the values 0·000126, 0·000160 and 0·000200 at temperatures −155°C., −64°C. and 23°C. respectively; while the author, for a rod of about the same dimensions, found an increase from 0·000224 at 18°C. to 0·000265 at 100°C. It seemed of importance therefore to attempt to discover whether this increase of heat loss is due to radiation alone, or to convection alone, or in part to both causes. In an attempt to clear up this point as far as possible, similar experiments to those described in the preceding section were performed at a temperature 100°C. Rods and wires of various size and material were tested, the table on the next page showing some of the results obtained.

1. *Total Losses of Heat from the Wire at* 100°C.

It will be seen from the results given that :—

(*a*) The percentage increase of loss of heat from the blackened wire over that from the bare wire is over 20, this being nearly double that under similar circumstances at air temperatures.

(*b*) The increase is about the same for all the metals tested.

(*c*) The diameter of the rod seems to have little or no appreciable influence on the result.

2. *Radiation Experiments at Various Temperatures.*

The " Leslie Cube " was filled with boiling water, and readings taken as before of galvanometer deflections when the cone

* C. H. Lees, " Bakerian Lecture," Roy. Soc. " Phil. Trans.," 1908.

(*a*) *Feb.* 4, 1915.—*Silver wire:* Diameter, 1 mm.; length, 35 cm.
Pt_1, 110·45°C.; Pt_2, 100·47°C.; "V," 10·18°C.

	Black.	Bright.	Wire out.	h_1/h_2.
E	7,535	7,452	6,562	1·202
C	390·4	386·4	340·7	

(*b*) *Feb.* 4, 1915.—*Copper wire:* Diameter, 1 mm.; length, 35 cm.
Pt_1, 110·63°C.; Pt_2, 100·47°C.; "V," 10·16°C.

	Black.	Bright.	Wire out.	h_1/h_2.
E	7,530	7,445	6 562	1·209
C	390·1	386·0	340·7	

(*c*) *Feb.* 8, 1915.—*Brass rod:* Diameter, 3 mm.; length, 30 cm.
Pt_1, 110·30°C.; Pt_2, 100·07°C.; "V," 10·23°C.

	Black.	Bright.	Rod out.	h_1/h_2.
E	8,570	8,390	6,517	1·213
C	443·0	435·0	338·5	

(*d*) *Feb.* 9, 1915.—*Bismuth rod:* Diameter, 6 mm.; length, 10 cm.
Pt_1, 110·00°C.; Pt_2, 99·89°C.; "V," 10·11°C.

	Black.	Bright	Rod out.	h_1/h_2.
E	7,643	7,546	6,475	1·214
C	396·9	391·5	336·3	

of the thermopile was placed at a fixed distance in front of (*a*) the metallic face, (*b*) the dead-black face of the cube. The readings were continued as the temperature of the water fell, this process being assisted by the addition from time to time of cold water. The galvanometer was suitably shunted at the various temperatures, so as to give readings on about the same part of the scale in each case. Cubes of brass and of "tinned iron" were employed, and gave results which did not differ materially from each other. Below are given readings in the case of the "tinned iron" cube:—

Radiation Experiments.

Feb. 11, 1915: "Tinned iron" cube, with one face dead-black.
Temperature of air, 13°C.; distance of galvanometer from scale, 2 metres.

Temperature of cube.	Deflection in millimetres.		Ratio.
	Black surface.	Metal surface.	
94°C.	433	82	5·28 : 1
65°C.	297	55·5	5·35 : 1
43°C.	206	39	5·29 : 1
32°C.	157	31	5·06 : 1
23·5°C.	220	43	5·11 : 1

Comparing these results with those previously given for a brass cube, it appears that the different metallic surface has very little influence on the result. Again, the ratio of emission of heat from the black surface to that from the metallic surface is very nearly the same at temperatures near 100°C. as at air temperatures. There is evidence, however, that this ratio (*i.e.*, the value of " b " of equation 2) increases slightly as higher temperatures are reached. This makes the value of " b " at 100°C. (for *brass*) 5·75—*i.e.*, 5 per cent. higher than at air temperatures. Also the value of " a " (from experiments 1 above) is 1·21.

Substituting these results in equations (4) and (5) we obtain :

$$\frac{r_2}{c}=\frac{a-1}{b-a}=\frac{0 \cdot 21}{4 \cdot 54}=4 \cdot 6 \text{ per cent.},$$

$$\frac{r_1}{c}=\frac{b(a-1)}{b-a}=26 \cdot 1 \text{ per cent.},$$

or the radiation from the bare wire at 100°C. is 4·6 per cent. of the convection, or 4·4 per cent. of the total loss of heat ; while that from the " black " wire is 26·1 per cent. and 20·7 per cent. respectively.

Also, from (6) and (7), *of a total loss of heat of* 100 *parts at* 100°*C.*, 4·4 *are due to radiation, and* 95·6 *to convection* (*for a " bare " wire*). *If the wire is coated with " dead-black," the corresponding percentages are* 20·7 *and* 79·3.

Not only, therefore, is the total amount of heat emitted greater at higher temperatures, but the proportion of radiation is greater. The percentage radiation loss of the total loss of heat rises from 2·5 per cent. to 4·4 per cent. for a rise of temperature from 12°C. to 100°C. For the same interval of temperature the increase in the total loss of heat is about 20 per cent. Hence, the radiation loss from a wire is increased in the proportion of 2·1 : 1 for the increase in temperature from 12°C. to 100°C., the excess of temperature of the wire being 10°C.

If we could assume the " Fourth Power " Law for this radiation loss, we should have (for an excess of 10°C. in each case)—

$$\frac{R_{12}}{R_{100}}=\frac{295^4-285^4}{383^4-373^4}=\frac{1}{2 \cdot 21}.$$

That is, the radiation loss at 100°C. would be (for " full " radiation) about 2·2 times that at 12°C., a result close to that obtained by experiment, which gave 2·1 : 1. Of course, this only indicates that the *ratio* of the radiations from the wire at

the two given temperatures is the same as the ratio of the "full" radiations at corresponding temperatures. It is clear, however, that the greater portion (nearly 90 per cent.) of the increase in the loss of heat from a wire at 100°C. above that at 12°C. is due to increased convection.

V. Discussion of Results.

It is interesting to compare the foregoing experimental results with those obtained from application of various radiation formulæ that have been proposed. For example, L. V. King (*loc. cit.*) has quoted a formula by Lummer and Kurlbaum for the radiation loss from a platinum surface, $e=0{\cdot}514\,(\theta/1{,}000)^{5\cdot2}$ watts per square centimetre per 1°C. excess.

Taking the absolute temperature of the enclosure as 290°C. and that of the wire as 300°C., this gives

$e=0{\cdot}000038$ cals. per square centimetre for 10°C. excess.

Thus, for a wire of 1 mm. diameter, where the total loss of heat per square centimetre for 10°C. excess is 0·00533, we have

$$\frac{\text{Radiation loss}}{\text{Total loss}}=0{\cdot}7 \text{ per cent.}$$

In the case of a rod of 6 mm. diameter, the "total loss" under the same circumstances is only 0·000224, and here the loss by radiation would be 1·7 per cent. of the total loss. (The experiments of the present research give 2·5 per cent. in each case.)

Again, applying Stefan's "Fourth Power" law for full radiation, $H=\sigma(\theta^4-\theta_0{}^4)$, and taking $\theta=300$°C., $\theta_0=290$°C., and $\sigma=5{\cdot}35\times10^{-5}$ (ergs) (Valentiner, A. d. P., 1910),

$H=0{\cdot}001308$ cals. per square centimetre for 10°C. excess at air temperatures. (At 100°C. this would be doubled—*see* Section IV.)

Now, the present research gives (for a *black* surface in a *brass* enclosure),

12·6/100 of 0·00533=0·00067 calories radiated for 10°C. excess from a wire 1 mm. diameter—*i.e.*, about a half of what is required by Stefan's law. This is probably accounted for by the brass enclosure, and possibly also, partly, from the "black" varnish used being not exactly a "full" radiator.

It is noticeable in the experimental results of the present Paper that the proportion of radiation to convection is almost precisely the same for wires of all diameters. As the total "h" decreases with increase of diameter of the wire, this points to a decrease of actual radiation per square centimetre

as the diameter increases. I am not aware of any definite reason for this, but there is no doubt of the fact in the present arrangement of apparatus. It has been suggested that a considerable proportion of the heat lost from the wire when it is "blacked" may possibly be due to the increase of thickness of the "wire" caused by the coat of varnish. This is not very likely, however, for the coating was exceedingly thin, and, in addition, the thermal conductivity of the varnish is certainly very many times less than that of the metallic wire.

In conclusion, it is apparent that—at any rate in the case of a wire heated within an enclosure of limited dimensions—the radiation from a wire plays a more important part than has been supposed by some workers. On the other hand this loss is not sufficient, at ordinary temperatures at all events, to interfere seriously with the validity of Newton's "Law of cooling," unless the temperature of the wire is raised considerably above that of the enclosure.

Summary.

The object of the experiments here recorded was to determine the numerical relation between the radiation and the convection losses from a heated metallic wire or rod placed in a gas at constant temperature. The method consisted in (1) measuring the amount of heat required to maintain the temperature of the wire a given amount (about 10°C.) above that of the surrounding gas, the surface of the wire being (*a*) coated with a dead-black varnish, (*b*) uncoated; (2) comparing the radiations from two surfaces exactly similar to (*a*) and (*b*) by means of a thermopile.

It is shown that if the total heat lost from unit surface of the wire is "a" times greater from a "black" wire than from a "bare" one, while the radiation from the black surface is "b" times more that from the unblacked surface, then

$$\frac{r_2}{c}=\frac{a-b}{a-1}, \text{ and } \frac{r_1}{c}=\frac{b(a-b)}{a-1},$$

where r_2, r_1, are the radiations from "black" and "bare" surfaces respectively and c is the convection.

The experiments indicate that of 100 parts of "total heat" lost from a wire at air temperatures, 2·5 consist of radiation for a bare wire, and 12·6 for a "black" wire. At 100°C., these percentages become 4·4 and 20·7 respectively. The 20 per cent. increase in "total loss" of heat from 17°C. to 100°C. is thus caused chiefly by increased convection.

DISCUSSION.

Dr. H. Borns asked if the author had tested the validity of the assumption that the convection was independent of the nature of the surface of the wire.

Mr. F. E. Smith asked why the platinum thermometer employed to give the temperature of the enclosure was placed inside it. There would be a temperature gradient between the wire and the enclosure and the thermometer could not give the correct temperature of the latter.

Dr. W. Eccles asked if the size of the enclosure affected the results. He had once studied the effects of convection in lifting the boom of a small micro-balance. The convection was due to the heating effect of the current in the instrument and the magnitude of the effect depended on the size of the enclosure.

Dr. A. Russell welcomed the author's work as being the first experimental attempt to determine the proportion of heat lost by radiation and convection. In early discussions on the heating of wires prominence was given to the nature of the surface which they should have to make the heat lost as great as possible. It appeared from the author's results that practically all the heat was lost by convection, and so whether the surface was blackened or not was not of much consequence. With different diameters of wire, did the convection loss vary inversely as the square root of the diameter? It should do so if the convection currents were uniform. Did the amount of heat radiated per unit area depend on the curvature? From these results it seemed to decrease as the radius was increased.

Mr. J. H. Brinkworth (communicated remarks) referred to some results of his experiments by the continuous flow method on the specific heat of steam ("Phil Trans.," A. 535, pp. 434 and 436). In these measurements the heat loss from the calorimeter was varied by altering the pressure of the air in the sheath surrounding the flow tube. The heat loss per degree per centimetre length of flow tube, when the sheath was evacuated as perfectly as possible—in which case radiation alone occurred—lies between 5 and 10 per cent of the loss when the pressure in the sheath was about 1 mm. of mercury.

Dr. Barratt, in reply, said he did not quite see how to verify experimentally that the convection was the same with the two kinds of surface employed. He thought that if the surface was smooth in each case the convection loss was bound to be the same. The classic experiments of Dulong and Petit, however, led to the conclusion that the cooling effect of the gas was quite uninfluenced by a difference of surface of the hot body. With regard to the position of the second platinum thermometer, he had found that when it was placed about half-way along and below the wire it gave the temperature of the ascending gas, and it is this temperature that determines the rate of loss of heat by convection. The comparative thermal capacity of the jacket was so great that this temperature was almost exactly equal to that of the jacket. He had not tried the effect of different sized enclosures, and the results given are, of course, only strictly applicable to the particular case investigated. With wires of different diameters the total heat loss per square centimetre decreased as the diameter of the wire increased, but he had not seen whether or not the square-root law mentioned by the Chairman held for the convection loss. Mr. Brinkworth's results are important as showing that convection losses are predominant down to such a low pressure as 1 mm. of mercury.

II. *The Magnitude of the Thermal Resistance introduced at the Slightly Conical Junction of Two Solids, and its Variation with the Nature of the Surfaces in Contact.* By THOMAS BARRATT, *A.R.C.S.*, *D.Sc.*

RECEIVED JUNE 22, 1915.

1. *Introduction and Description of Apparatus.*

IN Papers on the thermal conductivity of solids, recently published,* it was necessary to determine the fall of temperature caused by the thermal resistance at the joint where the conical extremity of a solid rod fitted, as accurately as possible, into a hole bored in a solid copper cylinder. In the investigation there recorded the only case treated was where the joint was of brass-copper, and it was assumed that the magnitude of the effect would be the same under similar conditions for other solids. In order to test the validity of this assumption, similar experiments have been conducted to determine the variation, if any, of the thermal resistance with the nature of the surfaces in contact.

One end of a rod of brass, RR (*see* Figure on p. 2 in previous Paper), fitted accurately into a slightly tapering conical space bored in a solid cylinder of copper, C, which was enclosed in a hollow cylinder of thin copper. The latter could be heated to any required temperature (recorded by a platinum thermometer, Pt_1), by means of a coil of single silk covered pure platinum wire, gauge 30, which was closely wound round it. This coil again was covered by another cylinder of copper, AB, whose thickness increased from zero at one end to about 2 mm. at the end into which the brass rod RR was fitted.

The product of readings of the current (C) and E.M.F. (E) at the ends of the platinum coil round the copper cylinder gave the amount of heat necessary to maintain the temperature of the copper block C any given amount V above that of the constant temperature enclosure, the latter consisting of a water or steam-jacket, JJJJ, within which the whole apparatus was kept. The temperature of this enclosure was given by a second platinum thermometer Pt_2 A hollow cone of very thin brass was made to fit as accurately as possible the end of the rod, so as to form a "double joint," thus doubling the thermal

* T. Barratt, Phys. Soc. "Proc." XXIX, V., August 15, 1914; and XXVII, I, December 15, 1914.

resistance effect. Readings were then taken of the amounts of heat necessary to maintain a given temperature difference, V ($=Pt_1-Pt_2$), between the copper block and the enclosure :—

1. With the rod and hollow cone in position,
2. With the rod only in position,
3. With the rod and cone both removed.

If the difference of the amounts of heat in (2) and (3) be denoted by H, and that between (1) and (3) by H_1, it was shown in the former of the two Papers referred to above that

$$\frac{V}{V_1}=\frac{H}{H_1}=\frac{CE-C'E'}{C_1E_1-C'E'},$$

where V is the excess of temperature of the *copper block* over that of the enclosure.

V_1 the excess of temperature of the *hot end of the rod* over that of the enclosure.

H and H_1 the amounts of heat given to the rod with the cone "in" and "out" respectively.

C, E the current and E.M.F. with rod in and cone out.

C_1, E_1 the current and E.M.F. with rod and cone both in.

C', E' the current and E.M.F. with rod and cone both out.

Measurements made with the brass cone had indicated that the percentage fall of temperature at the junction was constant, within the errors of experiment, both for different excesses of temperature (V) and for actual temperatures (17°C. and 100°C.) of the enclosure.

The value of H/H_1 or V/V_1 was 1·025 as given by the mean of several concordant experiments.

II. Experiments with Different Surfaces in Contact.

This abrupt fall of temperature at a junction is no doubt mainly due to the film of air that must exist between the two surfaces in contact. In order to investigate the effect for other metals similar experiments were conducted employing the same apparatus, but with cones of various substances—*e.g.*, copper, aluminium, zinc, platinum, &c. In some experiments the end of the brass rod was coated with a very thin layer of copper, deposited electrolytically, this procedure having been followed in every case in the original experiments on the conductivity of metals. No difference whatever could be detected in the results, whether the end were "coppered" or not.

The "cones" employed were of thickness only one-tenth of a millimetre and were made by cutting out from the metal foil part of a sector of the exact size required, thus

and folding it over the conical end of the brass rod, so that AD lay alongside and just touching BC.

The usual procedure in a "full" experiment was as follows: The end of the brass rod of diameter 3 mm., length 30 cm., was fitted tightly into the junction, the current C turned on, and the apparatus left for one or two hours for temperature conditions to become steady. Readings were then taken of C, E, Pt_1 and Pt_2. The rod was quickly removed, one of the cones fitted to the end of it and replaced. When conditions were again steady (this requiring only a few minutes), C_1 and E_1 were read, after being adjusted to maintain V ($=Pt_1-Pt_2$) the same as at first. The other cones were fitted successively in the same manner, C' and E' were obtained with rod and cone both removed, and the cones were again tested, this time in reverse order. The means of the values of C_1 and E_1 for each cone were calculated, and taken as the true readings. The whole experiment, once temperature conditions were steady, took about 2 to $2\frac{1}{2}$ hours for the seven or eight cones, and it was usually found that very little variation of temperature occurred in that interval. Of course, the *difference* of temperature (Pt_1-Pt_2) was kept absolutely constant throughout.

Below are given the actual determinations in one of this series of experiments:—

December 31, 1914: Brass rod, end "coppered," diameter 3 mm. Temperature of "hot" end, 14·95°C.; temperature enclosure, 6·56°C.

No cone.	Cu.	Pt.	Zn.	Ag.	Al.	Sn.	Pb.	Brass.	Rod out.
E, 6,027	5,995	5,986	5,986	5,989	5,983	5,973	5,970	5,970	4,559
C, 3,861	3,844	3,841	3,841	3,844	3,843	3,838	3,840	3,843	2,926

For copper cone, $V/V_1=H/H_1=(CE-C'E')/(C_1E_1-C'E')=9{,}939/9{,}705 =1{\cdot}024$, $=2{\cdot}4$ per cent. fall of temperature at the junction.

Similarly, as calculated from observations above, the results for other cones were as follows:—

Pt.	Zn.	Ag.	Al.	Sn.	Pb.	Brass.
1·029 ...	1·029 ...	1·026 ...	1·029 ...	1·037 ...	1·035 ...	1·035.

The mean results of seven consecutive experiments were :—

Metal.........	Ag.	Cu.	Al.	Pb.	Sn.	Zn.	Pt.	Brass.
Fall of temp..	2·1%	2·1%	2·3%	3·1%	2·9%	2·7%	2·7%	3·0%

The mean percentage excess of V over V_1 for all the surfaces tested is thus very nearly 2·5 per cent., this being the value adopted in the Paper already referred to.

The value of V/V_1 is seen, however, to depend to some extent on the nature of the surfaces in contact. In fact, the thermal resistances at the junctions are found to be nearly in the same order as the reciprocals of the thermal conductivities (k) of the metal cones themselves. A comparison of "k" with the corresponding value of V/V_1 is given :—

Metal.	Ag.	Cu.	Al.	Zn.	Brass.	Pt.	Sn.	Pb.
"k"	1·006	0·918	0·480	0·265	0·260	0·166	0·155	0·083
V/V	2·1	2·1	2·3	2·7	3·0	2·7	2·9	3·1

III. Effect of "Coppering" the Joints, and of Smearing them with Olive Oil.

(*a*) It has been mentioned that no appreciable difference was observed in the value of V/V_1 (*i.e.*, of H/H_1) when the end of the brass rod was covered with a thin film of copper. To investigate this further, a brass rod of uniform diameter 3 mm. and length 35 cm. had both its ends turned down to exactly similar conical shape. Readings were then taken of the value of "H" (*i.e.*, CE) for a given temperature difference (V), when each end in turn was inserted in the copper block. The readings were alike to within 1 part in 1,000. One end was then "coppered" by electrolytic deposition, and the experiments repeated. The results were as follows :—

Brass end .... CE=2,855
Coppered end .... CE=2,852

indicating that the presence of the thin film of copper made no difference whatever to the thermal resistance at the junction.

(*b*) Several experimenters on thermal conductivity have smeared the junction of two metallic surfaces with grease or oil. This procedure was, therefore, tried, pure olive oil being employed. "H" was measured in the usual way, and then

"H" was determined when the joint was smeared with the olive oil. The results are given :—

January 11, 1915 : Pt_1, 14·63°C. ; Pt_2, 7·24°C. ; "V," 7·39°C.			
	Rod in, no oil.	Rod in, with oil.	Rod out.
Heat given	21,117	21,223	12,128

Hence, $H/H_1=V/V_1=9{,}115/9{,}069=1{\cdot}005$.

That is, the heat given to the rod is 0·5 per cent. greater when oil is put on the joint. In other words, the thermal resistance at the junction is 0·5 per cent. less *with* olive oil than without.

(In the original research, oil was not used at the joints, because it hindered necessary quickness in manipulation, and because of the danger of alteration of surface emission of heat if any oil were left on the rod itself, or if it oozed out of the joint.)

IV. Experiments with Rods of Low Thermal Conductivity.

In the research on the thermal conductivity of badly conducting solids (*loc. cit.*), the substances experimented upon were in the form of cylindrical rods about 6 mm. diameter. The thermal resistance at the junction in this case was not measured but was assumed to be the same for all the solids investigated as for a pure bismuth rod, when they were under precisely the same conditions. The thermal conductivity of the bismuth being assumed, it was unnecessary exactly to measure the fall of temperature at the joint, provided it could be assumed equal for all the solids examined. In order to test the truth of this assumption, similar experiments to those described above were carried out with the original bismuth rod. Cones of thin paper, tinfoil, copper, &c., were made to fit the joint, and the value of H/H_1 determined in the usual way. The following results were obtained in one of these experiments :—

January 20, 1915 : Bismuth rod, diameter, 6 mm. ; length, 10 cm. ; Pt_1, 24·79°C. ; Pt_2, 14·04°C. ; "V," 10·75°C.			
	Paper cone.	No cone.	Copper plug (in place of rod).
E	6,626	6,648	5,610
C	414·15	405·0	351·5

This gives $V/V_1=H/H_1=1{\cdot}019^*=1{\cdot}9$ per cent. excess.

* To obtain this result, 5 per cent. was added on to "H" and to "H_1," in order to account for the heat lost from the surface of the copper plug. The explanation of this correction has been given fully in one of the Papers mentioned above.

Similar experiments with a tinfoil cone gave $H/H_1=1{\cdot}008$ $=0{\cdot}8$ per cent. excess. With a copper cone the thermal resistance at the junction was less than 0·5 per cent.

The results indicate that in the case of thicker rods of badly conducting material the fall of temperature at the junction is rather less than in the case of thin rods of comparatively high conductivity. This result is somewhat unexpected, but the probable explanation is that the air space at the junction (of almost molecular dimensions) has a relatively smaller effect than with the thinner rods of higher thermal conductivity. Even when a Paper surface is used, the relative thermal resistance at the joint is very small (less than 2 per cent.), in spite of its extremely low thermal conductivity. When a metallic surface is fitted to the joint, very little difference indeed can be detected in the amount of heat H flowing into the bismuth rod.

V. *Interpretation of Results.*

The true explanation of these results may be that the thermal resistance at similar junctions of two solids is constant, and *entirely* due to the film of air present. (That is to say, there is no certain indication of any phenomenon analogous to the fall of electrical potential at the junction of two dissimilar metals, where the nature of the metallic surfaces determines the magnitude of the effect.) In the case of a metallic wire of diameter about 1 mm., the influence of this film of air is relatively large. The presence of the very thin "cones" of various metals causes an additional thermal resistance, the magnitude of which is small if expressed in terms of "air," and which varies with the thermal resistance of the metal of which the cone is composed. The latter conclusion is most strongly supported by the experiments on the thicker rods of low thermal conductivity, where the influence of the paper cone of high "air" equivalent is much more marked than that of a metallic cone.

Taken as a whole, therefore, the results obtained in the present series of experiments justify the assumptions made in former Papers, viz. :—

1. That the thermal resistance at junctions of various metals of comparatively high conductivity and small diameter (about 1 mm.) is constant, and has a value of $2\frac{1}{2}$ per cent.

2. That the thermal resistance at the junction of thicker rods (6 mm. diameter) of low thermal conductivity is rather

smaller, and does not depend on the material of which the rod is composed.

Summary.

The abrupt fall of temperature caused by the thermal resistance at the slightly conical junction of two solids has been examined, the method consisting in constructing a "double joint," thus doubling also the thermal resistance effect.

In the case of wires of small diameter, the fall of temperature was found to be $2\frac{1}{2}$ per cent.; that is, the temperature of the "hot" end of the wire is $2\frac{1}{2}$ per cent. lower than that of the copper block into which the end of the wire was fitted.

This percentage fall of temperature is practically the same at all temperatures of the enclosure (up to 100°C.), and is independent of the excess of temperature of the end of the wire above that of the enclosure (at any rate, up to 10°C. or 12°C.).

For wires of greater diameter (6 mm.) the resistance is rather less than for smaller wires.

The thermal resistance is probably entirely due to the thin film of air that must exist at the joint between the wire and the copper block.

The experiments have been carried out at the Wandsworth Technical Institute, and my thanks are due to the Principal and Governors thereof for their kind interest in the work.

III. *On the Determination, by the Method of Diffusive Convection, of the Coefficient of Diffusion of a Salt Dissolved in Water.*

By ALBERT GRIFFITHS, *D.Sc.* (*Manc.*), *A.R.C.S.* (*Lond.*), *Head of Physics Department, Birkbeck College, London.*

RECEIVED JUNE 10, 1915.

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§ 1. *Object and Theory of the Method.*

THE object of the research was to develop a method of determining the coefficient of diffusion of a salt dissolved in water which involves the application of purely physical, as distinct from chemical, measurements which possesses the characteristic that all measurements are made after the attainment of the steady state of diffusion, and which promises to possess the advantage that there shall be no disturbance of the apparatus, internally or externally, during the period of crucial observations.

Some of the essential features of the apparatus are sketched in Fig. 1.

A and B may be called the diffusion-tubes. Their upper ends terminate in cylindrical vessels which are connected together by the long capillary tube CD. The cylindrical vessels are each connected by two vertical tubes to vessels E and F. T is a large glass tank. The whole apparatus is first filled with water, and then the water in the tank is replaced by a solution of the substance whose coefficient of diffusion is desired. In general a flow of water takes place along the capillary; and if the rate of flow, when the steady state is

attained, be known, it is possible to calculate the coefficient of diffusion of the salt.

Let it be assumed that the area of the cross-section of the tube A is the same as the area of the cross-section of the tube B.

Let h_1=the height of the top of the tube B above the top of the tube A.

h_2=the height of the bottom of B above the bottom of A.

L_1=length of tube A.

l_1=distance from the top of A to a point in the tube A.

L_2=length of tube B.

l_2=distance from the top of B to a point in the tube B.

r=radius of capillary tube.

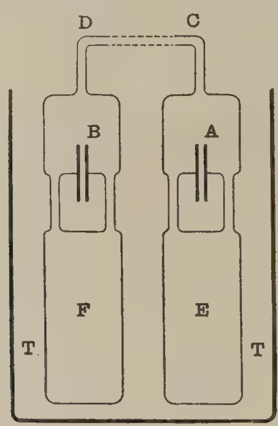


FIG. 1.

λ=length of capillary tube.

η=coefficient of viscosity of water.

q=ratio of area of cross-section of A or B to that of the capillary.

v_1=upward linear velocity of water at the top of the tube A.

v_2=downward linear velocity of water at the top of the tube B.

V=average linear velocity of water along the capillary.

ρ=density of solution.

n=concentration of solution in grammes per cubic centimetre.

N=concentration of solution in the tank.

c_1=quantity of salt transmitted per second by the combined action of diffusion and convection across unit area of the tube A.

c_2=corresponding quantity for tube B.

k=coefficient of diffusion of the salt.

It is assumed that $\rho=1+an$, where a is a constant depending on the salt dissolved.

Let $b=1-a$.

Since, when the steady state is attained, the water crosses each section of the tube B at the same rate, it follows that

$$v_2=v''(\rho-n_2)=v''(1-bn_2),$$

where v'' is the downward velocity of the solution at a point in the tube where the concentration is n_2.

Similarly, under the same conditions, the mass of salt transmitted by the compound operation of convection and diffusion through each cross-section of B is constant, and

$$-v''n_2+\frac{kdn_2}{dl_2}=c_2\,;$$

whence
$$-\frac{v_2n_2}{1-bn_2}+\frac{kdn_2}{dl_2}=c_2.$$

A solution of the last differential equation is

$$\frac{-bn_2}{v_2-bc_2}+\frac{v_2}{(v_2-bc_2)^2}\log_e\frac{(v_2-bc_2)n_2+c_2}{c_2}=\frac{l_2}{k} \quad . \quad . \quad . \quad (1)$$

and
$$\frac{bn_1}{v_1+bc_1}-\frac{-v_1}{(v_1+bc_1)^2}\log_e\frac{c_1-(v_1+bc_1)n_1}{c_1}=\frac{l_1}{k} \quad . \quad . \quad . \quad (2)$$

The head, measured in centimetres of water, due to the liquid in the tube B, equals

$$\int_0^{L_2}\rho dl_2=\int_0^{L_2}(1+an_2)dl_2=L_2+a\int_0^{L_2}n_2dl,$$
$$=L_2+a(NL_2-\int_0^N l_2dn_2).$$

Equation (1) gives l_2 as a function of n_2, and the integration shows that

$$\int_0^{L_2}\rho dl_2=L_2+aNL_2-a\Big[kv_2\Big\{\frac{(v_2-bc_2)N+c_2}{(v_2-bc_2)^3}\Big\}\log\frac{(v_2-bc_2)N+c_2}{c_2}$$
$$-\frac{bkN^2}{2(v_2-bc_2)}-\frac{kv_2N}{(v_2-bc_2)^2}\Big]. \quad . \quad . \quad . \quad . \quad (3)$$

In a similar manner it can be proved that

$$\int_0^{L_1} \rho dl_1 = L_1 + aNL_1 - a\left[kv_1\left\{\frac{c_1-(v_1+bc_1)N}{(v_1+bc_1)^3}\right\}\log_e\left\{\frac{c_1-(v_1+bc_1)N}{c_1}\right\} + \frac{bkN^2}{2(v_1+bc_1)} + \frac{kv_1N}{(v_1+bc_1)^2}\right]. \quad (4)$$

Let z=the head in centimetres of water required to drive the water along the capillary with average linear velocity V, calculated from Poiseuille's equation.
Then

$$\int_0^{L_2} \rho dl_2 + h_2(1+aN) - \int_0^{L_1} \rho dl_1 - h_1 - z = 0. \quad (5)$$

It can easily be proved that the increase in the volume of a weak solution of the salt due to the addition of 1 gramme of the salt is numerically equal to b.

The volume-flow downwards through the upper section of the tube B is equal to the volume-flow along the capillary plus the increase in volume of the water of weak solution above B due to the salt transmitted along B.

Let S=the area of cross-section of tube A or B.

s=the area of cross-section of the capillary.

Then $\quad v_2S = Vs + bc_2S$ or $v_2 = \dfrac{Vs}{S} + bc_2$.

Let $\quad v = \dfrac{s}{S}V = \dfrac{V}{q}$.

It may be mentioned that v is the velocity there would be in the diffusion tubes if b equalled zero. It is approximately equal to the actual velocity of the liquid in the diffusion tubes.

Then $$v_2 = v + bc_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Similarly, $$v_1 = v - bc_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting these values of v_1 and v_2 in (1), (2), (3) and (4), and introducing limiting values when need be, the equations become

$$\frac{bN}{v} - \frac{v-bc_1}{v^2}\log_e\left\{1-\frac{vN}{c_1}\right\} - \frac{L_1}{k} = 0. \quad . \quad . \quad . \quad (8)$$

$$-\frac{bN}{v}+\frac{v+bc_2}{v^2}\log_e\left\{\frac{1+vN}{c_2}\right\}-\frac{L_2}{k}=0 \quad . \quad . \quad . \quad . \quad (9)$$

$$\int_0^{L_1}\rho dl=L+aNL-ak\left\{\frac{(v-bc_1)c_1\left(1-\frac{vN}{c_1}\right)\log_e\left(1-\frac{vN}{c_1}\right)}{v^3}\right.$$

$$\left.+\frac{bN^2}{2v}+\frac{(v-bc)N}{v^2}\right\}. \quad . \quad . \quad . \quad . \quad (10)$$

$$\int_0^{L_2}\rho dl=L+aNL-ak\left\{\frac{(v+bc_2)c_2\left(\frac{1+vN}{c_2}\right)}{v^3}\log_e\left(1+\frac{vN}{c_2}\right)\right.$$

$$\left.-\frac{bN^2}{2v}-\frac{(v+bc_2)N}{v^2}\right\}. \quad . \quad . \quad . \quad (11)$$

In one calculation the value of k was determined as follows :

An approximate value of k was assumed, and c_1 and c_2 were found by Horner's method. The values of

$$\int_0^{L_1}\rho dl \quad \text{and} \quad \int_0^{L_2}\rho dl$$

were then found, and the results substituted in equation (5). The value of the left-hand side of (5) did not equal zero. Another value of k was then taken and calculations made once more of the value of the left-hand side of (5). The value again did not equal zero. The true value of k was found by extrapolation.

When $L_1=L_2=L$, and $h_1=h_2=h$, as was the case in the experiments to be described, it can easily be proved that equation (5) simplifies to

$$\frac{ak}{v}\left\{(c_1+c_2)\frac{L}{k}+bN^2-2N\right\}+haN-z=0. \quad . \quad . \quad (12)$$

A more convenient form of equation (8) for purposes of calculation is

$$\varphi(c_1)=bNv-(v-bc_1)\log_e\left(1-\frac{vN}{c_1}\right)-\frac{L}{k}v^2=0. \quad . \quad . \quad (13)$$

and a more convenient form of (9) is

$$\varphi(c_2)=-bNv+(v+bc_2)\log_e\left(1+\frac{vN}{c_2}\right)-\frac{Lv^2}{k}=0 \quad . \quad . \quad (14)$$

On account of the utility of the functions in Horner's method of solving an equation by successive approximation it may be mentioned that

$$\varphi'(c_1)=b\log_e\left(1-\frac{v\mathrm{N}}{c_1}\right)-\frac{\mathrm{N}v}{c_1{}^2}\frac{(v-bc_1)}{1-\frac{v\mathrm{N}}{c_1}}, \quad . \quad . \quad . \quad . \quad (15)$$

and that

$$\varphi'(c_2)=b\log_e\left(1+\frac{v\mathrm{N}}{c_2}\right)-\frac{\mathrm{N}v}{c_2{}^2}\frac{(v+bc_2)}{1+\frac{v\mathrm{N}}{c_2}}. \quad . \quad . \quad . \quad . \quad (16)$$

An approximate value of k is first assumed, and c_1 and c_2 are determined by Horner's method. After a little experience approximate values of c_1 and c_2 may easily be obtained. At the commencement, approximate values of c_1 and c_2 are given by equations derived from (13) and (14) by putting b equal to zero. Precise values of c_1 and c_2 having been obtained, their values are substituted in equation (12).

The value of the left-hand side of (12) will not, in general, equal zero. Another value of k is then taken, c_1 and c_2 are determined once more, and the value of the left-hand side of (12) is again calculated. The true value of k is found by extrapolation.

Semi-Empirical Formulæ.—The calculations described in the preceding section are very tedious. The author some years ago did some work on the assumption that $d=1+n$; that is, on the assumption that $a=1$ and $b=0$.

The results then obtained are substantially true, in spite of the appreciably large value of b in the case of potassium chloride.

Let v_0 be the velocity of the water at the top of a diffusion tube. It can easily be proved that the velocity of the liquid at the bottom of the tube is $v_0\div(1-b\mathrm{N})$. The mean of the two velocities is

$$v_0\frac{1-\frac{b\mathrm{N}}{2}}{1-b\mathrm{N}} \quad \text{or} \quad \frac{v_0}{1-\frac{b\mathrm{N}}{2}}$$

approximately. By modifying the result which would be obtained on the simple assumption that $d=1+n$, and replacing

v_0 by $v \div \left(1-\frac{bN}{2}\right)$, the author has obtained the empirical formula

$$\varphi(x)=\frac{h}{L}-\frac{z}{aNL}, \quad \ldots \ldots \quad (17)$$

where
$$\varphi x=\frac{e^x+1}{e^x-1}-\frac{2}{x}=\frac{\cosh\frac{x}{2}}{\sinh\frac{x}{2}}-\frac{2}{x}. \quad \ldots \quad (18)$$

and
$$x=\frac{vL}{\left(1-\frac{bN}{2}\right)k}. \quad \ldots \ldots \quad (19)$$

It is comparatively easy to solve (17) by a method of interpolation ; having found (x), k follows by means of (19).

The empirical formula (17) gives k, in its application to the apparatus described in this Paper, to an accuracy of one-tenth per cent. ; and the author is inclined to think that it will apply to apparatus of considerably different dimensions.

Where the dimensions are those of the apparatus described in this Paper, (17) may be shown to be approximately equivalent to

$$k=\frac{vL\left\{1-\frac{3}{5}\left(\frac{h}{L}-\frac{z}{aNL}\right)^2\right\}}{6\left(1-\frac{bN}{2}\right)\left(\frac{h}{L}-\frac{z}{aNL}\right)} \quad \ldots \ldots \quad (20)$$

This also gives k to an accuracy of about one-tenth per cent. When the ratio of h to L is small and z is negligible (20) becomes

$$k=\frac{vL^2}{6h\left(1-\frac{bN}{2}\right)}. \quad \ldots \ldots \quad (21)$$

§ 2. *Notes on Earlier Work and Difficulties.*

In 1904 the author studied experimentally the relation between the velocity of the extremity of a solution of methyl violet along a capillary tube which originally contained water and the average rate of flow along the capillary ; and he determined the coefficient of diffusion of copper sulphate. The result was approximately correct, but the indication of the

speed given by the methyl violet was not sufficiently consistent to make the result of value. About this time the author's colleague, Mr. B. W. Clack, commenced some careful and painstaking work with potassium and sodium chloride, and the author thought it wise to work with these solutions. In the sequel the change of solution proved to be a source of increased difficulty. In 1910 the author developed a precise method of measuring the rate of flow along a capillary tube.*

Numerous experiments were performed with the diffusion-apparatus. The flow was always in the right direction, and was as a rule fairly constant during the journey of an index; but the flow was never the same for two experiments in succession until 1914, *when a radical change was made in the method of introducing the index.*

In 1911† it was discovered that fungoid growths formed in the capillary tube; but even after the elimination of the growths by the addition of a trace of a copper salt no two successive journeys gave the same speed.

Attention was finally directed to the method of introducing the index. This introduction was originally effected by a reservoir of about 30 cubic cm. capacity interposed in the course of the capillary tube, and in introducing the index the water in the reservoir was replaced by fresh water. The earlier forms of apparatus were designed so that the journey was completed in a week or less; but, by a fortunate chance, an apparatus was made in which the journey of the index lasted for a fortnight. It was discovered that the introduction of the fresh water always disturbed the flow, and that the disturbance was often appreciable for a week. Under similar conditions, the rate of flow during the second week attained to 5 or 10 per cent. the same value in successive experiments. *The author wishes to call special attention to this disturbance.* The head between the extremities of the capillary tube is, as a rule, of the order of one thousandth of a centimetre of water, and it is obvious that a slight resistance will create a large temporary disturbance. It may be mentioned that there is a disturbance even if the water that is taken out of the reservoir is returned to it. The author suspects the cause of the disturbance to be some lack of homogeneity in the water at the ends of the capillary tubes in the reservoir. The phenomenon

* "Proc." of Phys. Soc., Vol. XXIII., Part III., pp. 190-197. 1911.
† "Proc." of Phys. Soc., Vol. XXIV., Part V., pp. 350-357. 1912.

may be due to soluble impurities or to variations in the number of colloidal particles suspended in the water. Further research is desirable with apparatus not complicated by diffusion.

In the latest experiments a method of introducing an index has been employed which involves the substitution of only a small fraction of a cubic centimetre of water by a weak solution of uranine. The final form of the device was due to the author's wife. It involves the use of an ordinary glass tap-plug, and is described in a separate Paper. With it, successive indexes have approximately the same speed.

§ 3. *Details and Dimensions of the Apparatus.*

The measurements given are those used in the calculations, and they are not to be regarded as precise to the last significant figure.

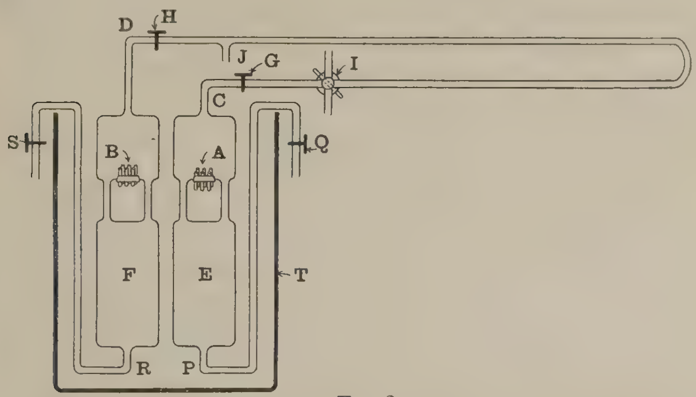


FIG. 2.

They are usually the result of averages, and the last figures are retained as tending to prevent the accumulation of small errors in the calculations.

The diffusion tubes A and B are composite in character, and each consists of eight tubes of average length 4·874 cms.; the average area of cross-section of each battery is 0·40515 sq. cm.

The capillary tube has an average internal diameter of 1·06 mm., and when calculating the head required to overcome the viscous resistance its length is taken to be 150 cm.

The glass vessels E and F, which may be called traps, because their function is to catch the salt transmitted along A and B, have each a capacity of 1,200 cubic cm.

The tank has a capacity of over 20 litres.

The capillary tube is provided with two main taps, G and H. The new arrangement for introducing the coloured index is suggested by I. (For the old method see Fig. 3, p. 196, "Proc." Phys. Soc., Vol. XXIII., Part III., 1911.) J is a vertical tube at right angles to the capillary. Tubes PQ and RS enable the traps to be filled and emptied. All taps and ends of tubes are provided with seals of liquid, generally mercury. The figure is diagrammatic, and in reality the two halves of the capillary tube are in the same horizontal plane.

§ 4. *Method of Conducting the Experiment.*

Distilled water to which a trace of copper chloride has been added was placed in aspirators and the dissolved air removed by connecting the aspirator to an exhaust pump. The copper chloride is added to kill or retard fungoid growths. Air-free water has two advantages :—

(*a*) Any small air bubble that has not been removed from the apparatus is speedily dissolved.

(*b*) Fungoid growths do not develop in freshly evacuated water—at any rate, in the author's experience.

After being stirred in a large carboy the water is added to the tank T and to the traps F and E. In order to prevent strain in the process of filling, care is taken to keep the levels of the water in F, E and T approximately the same. An exhaust pump is then attached to J and the whole apparatus filled with water. The apparatus is left to stand for a day or two. The main taps G and H are then closed and the tank is slowly filled with an evacuated solution of potassium chloride, to which a trace of copper chloride has been added.

The taps G and H are then opened, and after some days a coloured index is introduced into the circuit. The index liquid consists of an aqueous solution of uranine (generally $\frac{1}{4}$ gramme to the litre), to which a trace of copper chloride has been added. One cubic centimetre of a 20 per cent. solution of copper chloride was, as a rule, added to 20 litres of all the liquids employed.

§ 5. *References to Discarded Forms of Apparatus.*

Altogether about a dozen pieces of apparatus have been employed. Some of the devices were discarded in the attempt to avoid the mysterious disturbances, others were given up because of difficulties of construction. In one f

of the apparatus the tank was small, and the four short tubes leading to the traps F and E passed through the base of the tank. One of each pair of tubes proceeded to the bottom of the trap concerned. Each tube was provided with a three-way tap, so that it was possible to replenish the traps with pure water. It was hoped to analyse the liquid from the traps and to confirm the value of k found by the flow. In another form of the apparatus there was a device involving the use of a mercury seal, which enabled that part of the apparatus which holds the diffusion tube A to be adjusted by means of three screws.

Calculations of Results and Notes on Experiments.

It may be noted that, in the calculations to a first approximation, it is unnecessary to know the values of b, N and z, and that in calculations of precision it is unnecessary to know them with any extreme accuracy. Throughout the experiments b has been taken as 0·4058 and z as 0·000607 cm. of water. N was varied with the experiment.

§ 6. *Specimen Results with the Discarded Reservoir Method of Introducing Index.*

1. N=0·192 gramme per cubic centimetre, h=0·763 cm. The index was introduced 14 days after the filling of the apparatus. The first reading was taken a day later. In the table shown below the first column gives the hours elapsed after the first reading of the index, the second column gives the reading of the index, the third gives the speed of the index in the interval in centimetres per hour, the fourth gives the average speed after the first reading, the fifth gives the temperature in the interval.

Time in hours.	Reading in cms.	Speed in interval.	Speed from start.	Temperature in interval.
0	5·7	...	...	...
24·08	16·59	0·450	0·450	19·15°C.
48·08	27·78	0·466	0·458	19·05°C.
96·11	50·66	0·476	0·467	19·05°C.
168·08	85·55	0·485	0·475	19·09°C.
192·11	96·94	0·474	0·475	19·13°C.
216·08	108·61	0·487	0·476	19·14°C.
264·16	131·43	0·475	0·476	19·10°C.

A graph gives a convincing proof that there was a non-erratic action at work during the series of observations.

The speed at the end deduced from the tangent to the graph was 0·478 cm. per hour.

The temperature may be taken at 19·1°C.

The value of k obtained by means of (13), (14), (15) and (16) and (12) is $1{\cdot}552\times 10^{-5}$, and by means of (17), (18) and (19) it is $1{\cdot}550\times 10^{5-}$, and by means of (20) it is $1{\cdot}552\times 10^{-5}$.

A second index was introduced. The speed on the second day after the introduction of the index was 0·18 cm. per hour—*i.e.*, less than half what it would have been if no disturbance had been caused in the process of introducing the new index.

On the third day the speed was 0·44 cm. per hour, and it increased to 0·46 in the course of the following week.

2. N=0·203 grammes per cubic centimetre. h=0·821 cm.

The speed was between the limits 0·43 cm. and 0·54 cm. per hour.

If the hypothesis be made that the disturbance is statical and not dynamical in character, then the best value of the speed is the highest—viz., 0·54 cm. per hour. The temperature may be taken as 18·9°C., the value of k from (12), &c., is $1{\cdot}628\times 10^{-5}$, from (17), &c., it is $1{\cdot}629\times 10^{-5}$, from (20) it is $1{\cdot}629\times 10^{-5}$. *This result agrees to 2 per cent. with the value given with the superior method of introducing the index.*

§ 7. *Experiments and Results with the Successful Tap Plug Method of Introducing Index.*

By introducing a second index whilst a first was in the middle of its journey it was proved conclusively that the introduction of the index does not produce a large disturbance. The readings suggest that in general the introduction of a second index increases the reading of the first index by 1 mm. The effect may be due to mechanical disturbance in the process of introducing the second index. The tap, made abroad, was not according to specification, and the index was long and diffuse and not symmetrical. Readings were made of the two ends and the average taken. Mrs. Griffiths, Mr. H. G. Bell and the author took independent readings of an index, and the calculated speeds during one experiment were 0·537, 0·537 and 0·540 respectively, so that, in spite of the poor index, the results were consistent. After the improved method of reading a bad index, described in a following Paper, had been evolved, it

was proved that the earlier method of taking the reading gives substantially accurate results.

An attempt was made to determine k for weak solutions ; but, unfortunately, various deposits had formed and adhered to the glass in various parts of the interior of the apparatus ; and as, on account of the cements employed in putting the apparatus together, an attempt to clean the interior only made matters worse, the instrument had to be dismantled.

The variety and manner in which growths form in the apparatus is annoying and remarkable. A white growth frequently formed in the vertical tube J. An expert chemist thought the growth was botanical, whilst an expert botanist thought it was chemical. A white curdy deposit often formed near a tap even when no lubricant was used. Over long periods the use of a copper salt for keeping down fungoid growths is not very satisfactory. The salt appears gradually to come out of solution, perhaps as an oxy-salt ; at any rate, living fungoid growths do sometimes appear in the solution after some weeks.

The results given by experiments in which the conditions were kept constant throughout the long series of observations are set out in the following table. ${}_0K_N$ is given in C.G.S. units.

	Diff. of level.	Speed in cms. per hour.	v.	N.	Temp.	${}_0K_N$.
1	0·788	0·535	$3{\cdot}244 \times 10^{-6}$	0·213	20·0	$1{\cdot}691 \times 10^{-5}$
2	0·788	0·542	$3{\cdot}286 \times 10^{-6}$	0·213	20·2	$1{\cdot}710 \times 10^{-5}$
3	0·788	0·539	$3{\cdot}268 \times 10^{-6}$	0·213	20·3	$1{\cdot}700 \times 10^{-5}$
4	0·788	0·538	$3{\cdot}262 \times 10^{-6}$	0·210	20·2	$1{\cdot}696 \times 10^{-5}$

The results of the above table give a mean value of ${}_0K_N$, equal to $1{\cdot}70 \times 10^{-5}$ at a temperature of 20·2°C. in the case of an aqueous solution of potassium chloride containing 0·21 grammes of salt to the cubic centimetre. This is 6 per cent. higher than that obtained by the author's colleague, Mr. B. W. Clack. The difference is small compared with the differences between the published results in diffusion ; but it is probably just outside the errors of observation and further research is necessary.

In conclusion, the author would like to express his indebtedness to Mr. W. E. Paterson, M.A., B.Sc., Rev. M. Collett, B.Sc., Mr. J. M. Dickson, B.Sc., Mr. M. C. Boff, Mr. H. G. Bell and Mrs. C. H. Griffiths, B.Sc., for much painstaking help in very monotonous tasks.

N.B.—In a previous Paper by the author, "Proc." Phys. Soc., XVI., Part IV., pp. 230–243, the following errors occur :

Page	235	*line*	23,	5·09	*should be*	0·727
,,	238	,,	14,	5	,,	0·7
,,	242	,,	12,	0·019	,,	0·19
,,	242	,,	13,	0·08	,,	0·8

ABSTRACT.

In general the diffusion of matter like the diffusion of heat produces convective currents, and there is a diffusive convection which is akin to thermal convection. In the apparatus described in the Paper the convective flow is of the order of one millionth of a cubic centimetre per second from the top of one set of diffusion tubes to the top of another set. The top of each set of diffusion tubes is enclosed in a glass vessel containing water, and the one vessel is connected to the other by means of a long capillary tube. Each set of eight diffusion tubes is of length about 5 cm., and has a total area of cross-section of about 0·5 sq. cm. The capillary tube is about 150 cm. long, and has a diameter of about 1 mm. ; and the linear flow of a coloured index, consisting of a feeble solution of uranine, is of the order of 10 cm. per day. The index is introduced by means of a special four-way glass tap.

DISCUSSION.

Mr. B. W. Clack said he had repeated his experiments under all possible conditions to try to locate the cause of the 6 per cent. difference between his results and those of the author, but so far without success. In the author's arrangement the water was inside the tube and the solution was in the tank. He had tried that with his apparatus but gave it up in 1908 in favour of having the water in the tank and the solution inside. He showed some curves taken with the two arrangements. In the case of dilute solutions the results were the same, but with strong solutions there was about 4 per cent. difference. It may be in this direction that the explanation of the discrepancy has to be looked for.

Dr. T. Barratt asked if the introduction of a salt to prevent the formation of fungoid growths would not be likely to cause some error.

Dr. Griffiths replied that the amount of copper chloride introduced was exceedingly small, about 1 cubic cm. of 20 per cent. solution being added to the liquid in question, and he did not think it could produce any appreciable error.

IV. *The Effect of Electric Oscillations on the Magnetic Properties of Iron investigated by the Campograph.* *By* J. A. FLEMING, *M.A., D.Sc., F.R.S., and* P. R. COURSEY, *B.Sc.*

RECEIVED OCTOBER 25, 1915.

IN a Paper read to the Physical Society in March, 1915, by one of us (Dr. J. A. Fleming), a description was given of an instrument invented by him for delineating physical curves which he called a campograph.* Since that date great improvements have been made by him in the instrument by the substitution of mirrors of worked glass silvered on the front for the common silvered-on-the-back mirrors first employed which has resulted in a great optical improvement and in a resulting fineness in the lines on the photographic plates. The object of the following research was to apply this improved instrument in the study of the effects on the hysteresis curves of iron of superimposed oscillations or currents through the iron. For the arrangements adopted we refer to the Paper of Dr. Fleming (*loc. cit.*). Suffice it to say that a new magnetising helix of 71,136 turns was constructed 1 metre in length and the iron wires employed were of 50 cm. in length and about 2 mm. in diameter. Hence the iron may be regarded as placed in a nearly uniform field. We refer also to the above-mentioned Paper for details as to the mode of calibration of the campograph so as to determine the absolute value of the hysteresis from the photographic delineation of the curves.

The iron wire employed was placed in this long magnetising helix and fixed in position near the magnetometer and campograph. It was then easy to photograph a hysteresis loop for the iron by slowly varying in a cyclical manner a direct current through the magnetising helix. The illustrations in this paper are reproduced from the actual negatives, each taken in a very few minutes. Also at the same time alternating currents or high-frequency oscillations could be passed through the iron wire, or made to circulate round it, through an inner helix wound on the iron wire. This last coil had 800 turns of No. 26 S.W.G. copper wire, and the length of the helix was 46·5 cm. The R.M.S. value of the electric oscillations damped or undamped which passed through it was measured by a thermo-electric ammeter.

* "An Instrument for the Optical Delineation and Projection of Physical Curves," by J. A. Fleming, "Proc." Phys. Soc. Lond., Vol. XXVII., p. 316, 1915.

These dispositions being made we proceeded to study the effect on the magnetic hysteresis of iron of electric oscillations flowing around the wire or else along it, whilst at the same time the iron was subjected to a slowly varying cyclical magnetising force under various conditions.

1. The first group of observations were on the effect of damped electric oscillations of constant R.M.S. value (about 70 milliamperes) and constant oscillation frequency (about $0 \cdot 75 \times 10^6$), and spark frequency (about 250), but the maximum value of the slow cyclical magnetising force varying from about $H_{max.}=1$ to $H_{max.}=10$. This effect is different for small forces and large. If $H_{max.}$ lies between about 0·2 and 3 C.G.S. the effect of the oscillations is to produce a very marked increase in the hysteresis and a marked increase in the magnetic flux corresponding to the maximum force, with a maximum effect in the neighbourhood of $H=0 \cdot 5$ to $0 \cdot 7$. If $H_{max.}$ lies between 3 and 6 then the effect is in the same direction but less marked. If, however, $H_{max.}$ is about 8 or 10 then the hysteresis loop becomes smaller by the corners being rounded off. These various effects are shown by the photographs in Fig. 1, 2 and 3. Hence, for small slowly cyclical magnetising forces the effect of superimposing electric oscillations is greatly to increase the hysteresis loss and also the magnetic flux density corresponding to the maximum force. For large maximum cyclical forces the effect of the oscillations is to diminish the flux at or near the zero value of the magnetising force. Since the coil carrying the oscillations had 800 turns and a length of 46·5 cm. it will be seen that the turns per centimetre are 17·2, and hence the magnetising force for 70 milliamperes is $H=1 \cdot 5$.

For the damped oscillations used in this set of experiments the ratio of the maximum value of the current in a train to the R.M.S. value was very nearly 100. Hence the magnetising force rose at some instants to nearly 150, but had an R.M.S. value of 1·5. It follows, therefore, that the oscillatory magnetic force carried up the flux density to saturation at certain instants 250 times a second. The explanation of the above effects may be as follows: Superimposed upon the slowly cyclical force we have a rapidly cyclical force of very varying maximum. The effect of this is to shake up the magnetic molecules of the iron and to promote greater responsive magnetisation to the slow cyclical force at the extreme ends of the cycle, but this action also increases the retentivity and, therefore, the hysteresis. On the other hand, for large values of

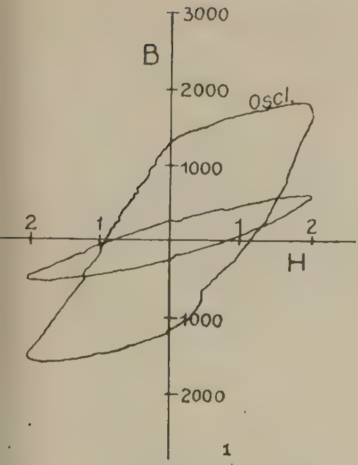


FIG 1.

.M.S. current $=70\times10^{-3}$amp.; frequency= 5×10^6; spark frequency=250 per second; de- nent=0·68.
Iysteresis loss per cm.^{3} per cycle: normal 17 ergs; with oscillations=482 ergs.
Iaximum flux density=600 (normal), with llations=1,730.

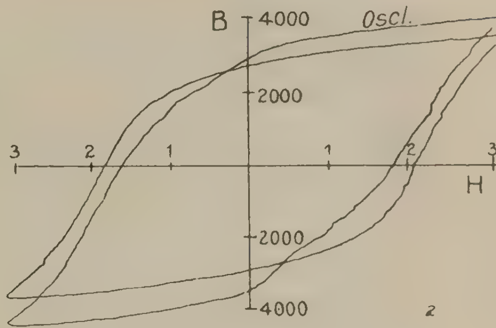


FIG. 2.

Current $=70\times10^{-3}$amp.; frequency $=0{\cdot}75\times10^6$; spark frequency=250 per second; decrement =0·68.
Hysteresis loss per cm.^{3} per cycle: normal =1,800 ergs; with oscillations=1,810 ergs.
Maximum flux density: normal =3,470; with oscillations=4,190.

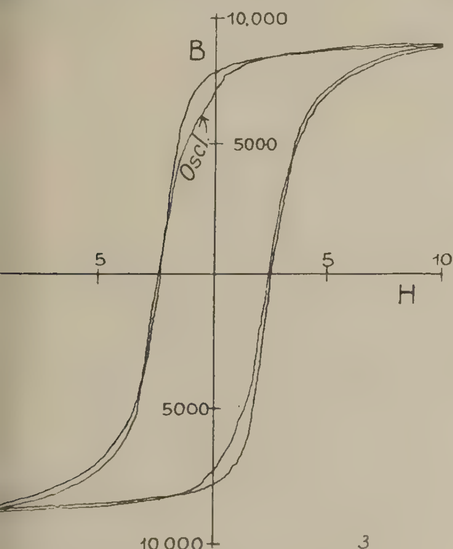


FIG. 3.

R.M.S. current $=70\times10^{-3}$ amp.; frequency $=0{\cdot}75\times10^6$; spark frequency=250 per second; decrement=0·68.
Hysteresis loss per cm.^{3} per cycle: normal =7,360 ergs; with oscillations=6,770 ergs.
Maximum flux density: normal=8,820; with oscillations=8,900.

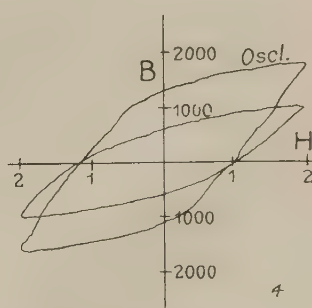


FIG. 4.

R.M.S. current $=57\times10^{-3}$ amp.; frequency $=0{\cdot}75\times10^6$; spark frequency=250 per second decrement=0·68.
Hysteresis loss per cm.^{3} per cycle: normal =250 ergs; with oscillations=490 ergs.
Maximum flux density: normal=840; with oscillations=1,450.

the slowly periodic force the effect of the oscillations is somewhat to reduce retentivity and so reduce the magnetisation at the moments of zero force and, therefore, to cut off the corners of the hysteresis loop.

2. In the second set of experiments the maximum value of the slowly cyclical force was kept constant at $H=2$ and the oscillations at the constant frequency of $0{\cdot}75 \times 10^6$ and spark frequency at 250, but the R.M.S. value of the oscillations varied from 23 to 57 milliamperes, and hence their magnetic force in the same ratio. It was found that the effect of the oscillations was to increase the area of the hysteresis loop almost in a linear relation to the oscillatory currents and also to increase the maximum value of the flux density corresponding to maximum of the slowly cyclical force. (*See* Fig. 4.)

3. In the third set of experiments the slowly varying magnetising force was kept within limits $H_{max.}=2$ and the oscillatory current kept of constant R.M.S. value (60 milliamperes), but its oscillation frequency varied from $0{\cdot}75 \times 10^6$ downwards. The spark frequency was kept at 250 for the high-frequency damped oscillations. In this case there was a very marked increase in both the area of the hysteresis curve and in the maximum value of the flux density (B) corresponding to the maximum slowly varying magnetising force.

The photographs in Figs. 5 and 6 show the effects when the oscillation frequency was 100,000 and 11,000 respectively, the R.M.S. value of the oscillations being in both cases 60 milliamperes and the spark frequency 250. The decrement in the case of Fig. 5 was 0·265 and for Fig. 6, 1·07. The superior effect of the lower frequency is no doubt due to the greater depth of penetration of the low-frequency flux into the iron. The same type of effect takes place if the oscillations are replaced by an undamped high-frequency current from an alternator. Fig. 7 shows the result when persistent oscillations of frequency 2,800 ∽ were employed, and it was most marked when the frequency of the alternating current was reduced to 100 ∽. Fig. 8 shows the remarkable effect of carrying an iron wire through a slowly varying magnetic cycle ($H_{max.}=2$), whilst at the same time the iron is subjected to the action of an alternating magnetising force of frequency 100 ∽. The maximum flux density is immensely increased. There is a marked difference between the effect of undamped and damped oscillations of the same R.M.S. value and frequency. Thus Fig. 9 shows the result of superposing an alternating current of 45

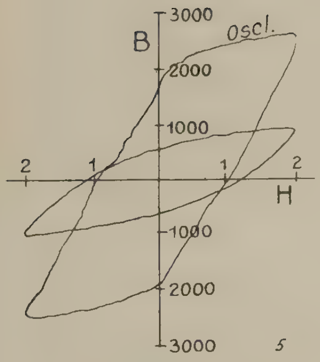


FIG. 5.

R.M.S. current $=60\times10^{-3}$ amp.; frequency $=10^5$; spark frequency $=250$ per second; decrement $=0{\cdot}26$.

Hysteresis loss per cm.^{3} per cycle: normal $=262$ ergs; with oscillations $=675$ ergs.

Maximum flux density: normal $=945$; with oscillations $=2{,}520$.

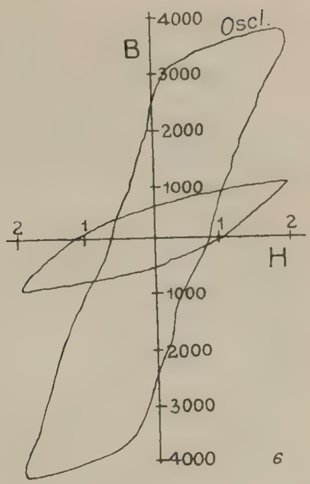


FIG. 6.

R.M.S. current $=60\times10^{-3}$ amp.; frequency $=11{,}000$ spark frequency $=250$; decrement $=1{\cdot}07$.

Hysteresis loss per cm.^{3} per cycle: normal $=231$ ergs; with oscillations $=885$ ergs.

Maximum flux density: normal $=960$; with oscillations $=4{,}050$.

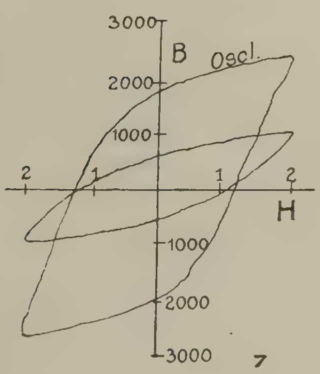


FIG. 7.

R.M.S. current $=60\times10^{-3}$ amp.; frequency $=2{,}800$; decrement $=0$.

Hysteresis loss per cm.^{3} per cycle: normal $=263$ ergs; with oscillations $=760$ ergs.

Maximum flux density: normal $=965$; with oscillations $=2{,}480$.

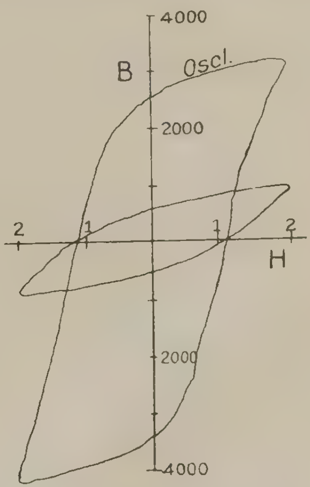


FIG. 8.

R.M.S. current $=60\times10^{-3}$ amp.; frequency $=100$; decrement $=0$.

Hysteresis loss per cm.^{3} per cycle: normal $=239$ ergs; with oscillations $=1{,}150$ ergs.

Maximum flux density: normal $=930$; with oscillations $=3{,}720$.

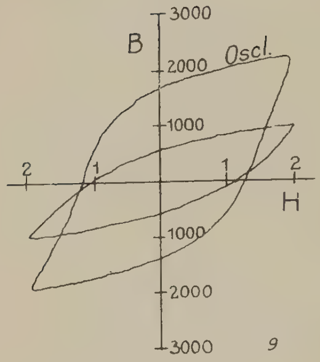


FIG. 9.

R.M.S. current=45×10⁻³ amp.; frequency =4,500; decrement=0.

Hysteresis loss per cm.³ per cycle: normal=256 ergs; with oscillations=655 ergs.

Maximum flux density: normal=972; with oscillations=2,050.

Compare with Fig. 10.

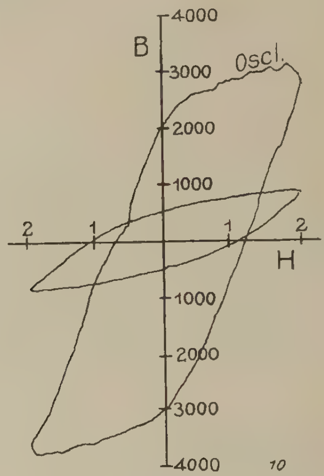


FIG. 10.

R.M.S. current=45×10⁻³ amp.; frequency =4,500; spark frequency=250 per second; decrement=0·51.

Hysteresis loss per cm.³ per cycle: normal =228 ergs; with oscillations=912 ergs.

Maximum flux density: normal=905; with oscillations=3,420.

Compare with Fig. 9.

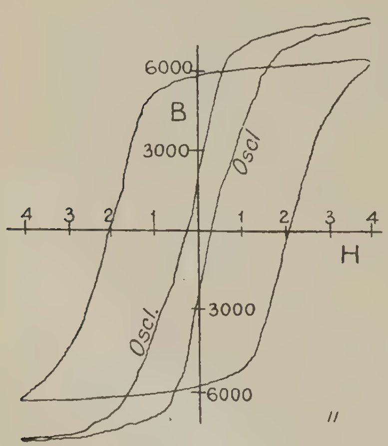


FIG. 11.

R.M.S. current=9·5 amps. along the wire; frequency=186,000; spark frequency=200; decrement=0·209.

Hysteresis loss in ergs per cm.³ per cycle: normal =4,110; with oscillations=1,010.

Maximum flux density: normal=6,320; with oscillations=7,900.

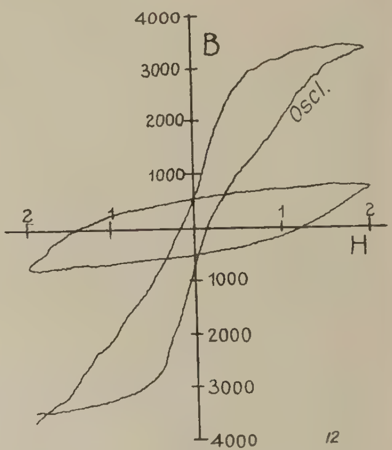


FIG. 12.

R.M.S. current=9·5 amps. along the wire; frequency=186,000; decrement=0·209; spark frequency=abt. 200.

Hysteresis loss in ergs per cm.³ per cycle: normal =250; with oscillations=282.

Maximum flux density: normal=778; with oscillations=3,460.

milliamperes and frequency 4,500 on a slowly varying magnetic cycle, whilst Fig. 10 shows the effect of using a damped oscillation of decrement 0·51, but the same frequency and R.M.S. value. The increase in maximum flux density produced by the damped oscillations is greater than by the undamped. Since in the latter case the ratio of maximum to R.M.S. value is much greater, this seems to indicate that the maximum value of the force is effective.

4. In the fourth set of experiments the effect of sending damped or undamped oscillations of various strengths and frequencies along an iron wire, on the magnetic hysteresis curve due to slowly varying force was investigated.

In the first place the oscillations used were damped oscillations of R.M.S. value 0·58 ampere, frequency $0{\cdot}488 \times 10^6$ and decrement 0·172. The maximum value of the slowly cyclical magnetising force varied from $H_{max.}=2$ to $H_{max.}=10$ in different experiments.

The effect was found to be generally much the same as when the oscillations passed round the wire, viz., for small maximum value of H the hysteresis loop was increased and for values of $H_{max.}=8$ or 10 the area was diminished by the shoulders of the curve being rounded off.

In the second set of experiments an oscillatory current of 9·5 amperes was used, and the frequency lowered to 186,000 and decrement was 0·209. The spark frequency was about 200.

In a third set of experiments the oscillatory current was carried up to 20 amperes, at an oscillation frequency of 477,000 and decrement of 0·156. When the slowly varying magnetic force had a maximum value of about 3 to 4 or more the effect of the oscillations in this case was to close up or diminish the area of the hysteresis loop by an amount depending on the current strength, but at the same time it increased the maximum value of the flux density very greatly for small maximum values of the slowly varying cyclical magnetising force. (*See* Figs. 11, 12 and 13.)

There is, therefore, a double action. The hysteresis is diminished, but the magnetisation is increased at the extreme ends of the cyclical curve, except when the current is very large when the extreme magnetisation is diminished. This diminution of the hysteresis is also found when a strong continuous current is sent along the wire. It is not an effect due

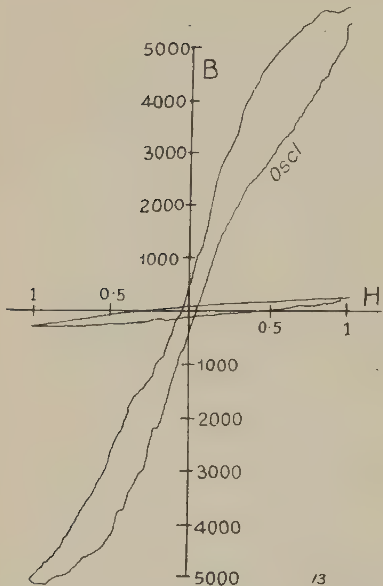


Fig. 13.

R.M.S. current=9·5 amps. along the wire; frequency=186,000; spark frequency=abt. 200; decrement=0·209.

Hysteresis loss per cm.³ per cycle, in ergs: normal =24·3; with oscillations=195.

Maximum flux density: normal=286; with oscillations=5,440.

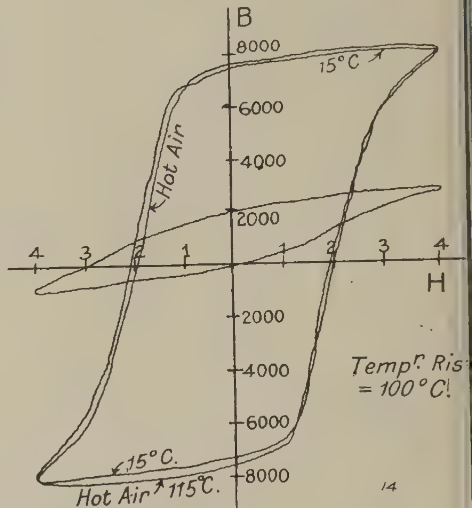


Fig. 14.

Direct current=15 amps. along wire.

Hysteresis loss per cm.³ per cycle: normal =5,160 ergs; with hot air=5,200 ergs; with current=850 ergs.

Maximum flux density: normal=8,150; with hot air=8,150; with current=1,860.

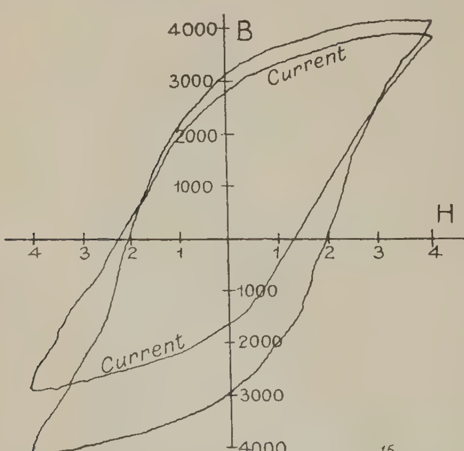


Fig. 15.

Direct current along tube=15 amps.

Hysteresis loss in ergs per cm.³ per cycle: normal =2,285; with current=1,715.

Maximum flux density: normal=4,140; with current=3,360.

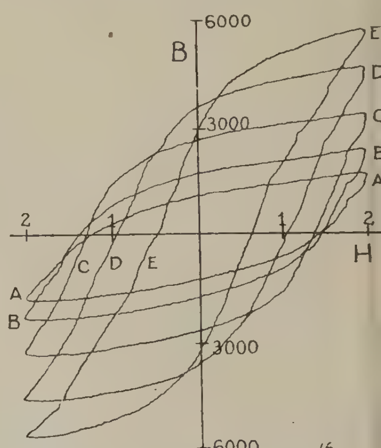


Fig. 16.

Frequency=80 ∽; decrement=0.

Curve.	Current.	Loss in ergs.	Bmax
A	0	475	1,760
B	0·25	813	2,380
C	0·50	1,190	3,400
D	0·75	1,340	4,580
E	1·0	976	5,640

Current along iron wire.

to heat.* This was proved by sending along the iron wire a current of hot air. In other words, when the wire was heated by hot air to about the same temperature (115°C.) that it was heated by the electric current sent along it, no sensible diminution in area of the hysteresis loop was found. (*See* Fig. 14.)

It is clear, however, that this diminution of the hysteresis, as shown by the smaller loop in Fig. 14, is the result of the strong circular magnetisation produced by the current, whether direct or alternating, flowing along the wire. It is well known that the hysteresis loss of iron is reduced to a small value when the iron is rotated in a very strong magnetic field. If, then, a strong current flows along a rather thin iron wire producing a circular magnetisation, the corresponding circular magnetic flux may reach a high value, and the result of superimposing a slowly cyclical longitudinal magnetisation may be that the hysteresis loss due to the latter is greatly reduced. The magnetic force just at the surface of the iron wire is $A/5r$, where A is the current conveyed in amperes, and r is the radius of the wire in centimetres. In this case $A=15$ nearly and $r=0{\cdot}08$ cm. Hence the force is about 37 C.G.S. units.

There is accordingly a strong circular magnetisation, and this grips the molecules so that it reduces the hysteresis under longitudinal force. Moreover, when the current is very strong it reduces the longitudinal magnetisation. This view was confirmed by using a thin walled tube of iron instead of a solid wire, the total cross-section of iron and the current flowing through it being the same in both cases. The two diagrams in Figs. 14 and 15 show the two cases of a solid iron wire and an equi-sectional iron tube of overall diameter of 1 cm. It is clear that in the case of the tube the circular magnetising force at the surface is much less than in the case of the thin wire for equal current conveyed. Hence we find that in the case of the tube there is a much less reduction in the area of the hysteresis loop due to slowly varying longitudinal force. The above explanation therefore seems valid.

When the longitudinal cyclical force has a small maximum value $H=1$ the effect of sending strong oscillatory currents along the wire is to produce a remarkable increase in the

* In the Paper by Dr. J. A. Fleming describing the campograph, *see* "Proc." Phys. Soc., Lond., June, 1915, in Fig. 8 of that Paper the reduction in area of a hysteresis loop due to the passage of a current along the wire is erroneously attributed to heat, whereas it is, in fact, due to circular magnetisation.

maximum flux density at the extreme ends of the cycle. It also increases the area of the loop, as shown in Fig. 13. The superimposed oscillations then promote magnetisation by shaking up the magnetic molecules both when the electric oscillations flow along the wire and when they flow round it through an insulated helix.

If, then, we send along the wire various oscillatory currents at constant frequency, but at the same time vary the maximum value of the slowly cyclical magnetising force, the result is that the hysteresis loop is increased for small maximum values ($H=1$) of H, but diminished for larger values ($H=4$ to 8), but

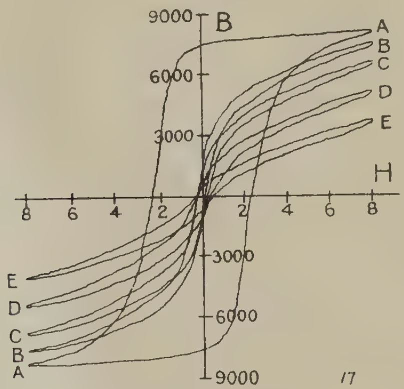


FIG. 17.

Frequency=80 ∽; decrement=0.

[Curve.	Current.	Loss in ergs.	B_{max}.
A	0	6,650	8,220
B	4·5	753	7,650
C	7·0	615	6,750
D	12·0	684	5,250
E	20·0	615	3,920

Current along iron wire.

the maximum value of the flux density at the ends of the loop is always increased, except in the case of very large values of $H_{max.}$ and of current, when there is a decrease. The oscillations, therefore, increase the apparent permeability at the maximum magnetisation when $H_{max.}$ is less than 8 but slightly decrease it for larger values.

5. The effect was next studied of sending a low frequency ($n=80$ ∽) alternating current along the wire, whilst at the same time a slowly varying cyclical longitudinal magnetising force was applied to the wire. Out of a large number of photographs two are selected which show the results for $H_{max.}=2$

and $H_{max.}=8$, and for currents varying from 0·25 ampere to 20 amperes. (*See* Figs. 16 and 17.)

In these diagrams the curves marked A are the original hysteresis loops, and the other curves B, C, D, E are the loops when the longitudinal currents of the strength stated below the diagrams flow along the wire. Fig. 16 shows the increase in apparent permeability at the ends of the cycle, and Fig. 17 the reduction both in hysteresis and longitudinal permeability for strong currents due to the circular magnetisation.

6. To extend these results the campograph connections were

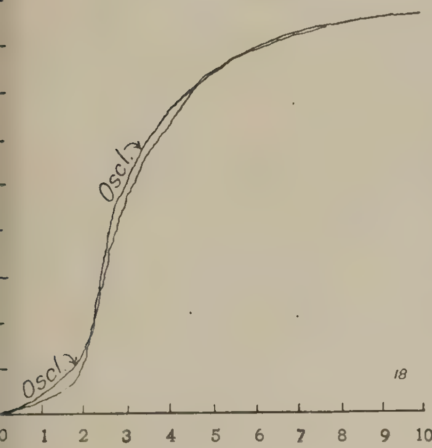


FIG. 18.

R.M.S. current$=60\times10^{-3}$ amp.; frequency $=0{\cdot}75\times10^{6}$; decrement$=0{\cdot}68$.
Spark frequency$=250$ per second.

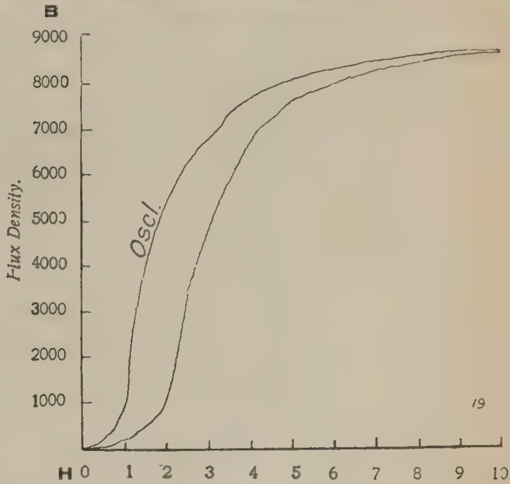


FIG. 19.

R.M.S. current$=60\times10^{-3}$ amp.; frequency $=100$; decrement$=0$.

re-arranged so as to delineate an ordinary magnetisation (non-cyclical) curve. The magnetisation curve was then taken with and without an oscillatory current of 60 milliamperes at a frequency of $0{\cdot}75\times10^{6}$ and decrement$=0{\cdot}68$ flowing round the wire. The result is shown in Fig. 18, by which it is seen that the effect is slightly to increase the flux at two points on the magnetisation curve.

In the next case the magnetisation curve was photographed with and without a persistent alternating low-frequency current ($n=100$) of 60 milliamperes flowing round the wire. The effect is shown in Fig. 19, and is seen to be a great increase in flux density all along the curve. For magnetising forces near $H=2$ the flux density is increased about five times, as

shown in Fig. 20, which gives the permeability curves deduced from Figs. 18 and 19.

Conclusions.—The photographs reproduced in this Paper are only a few out of a very large number taken, which have all been carefully analysed. Broadly and generally, the effect on an iron wire of a longitudinal slowly varying cyclical magnetising force, on which an alternating magnetisation, also longitudinal or else circular, is superimposed, is as follows: When the cyclical magnetic force has a small maximum (H=1 about) the effect either of superimposed damped or undamped oscillations of high or low frequency is to increase the hysteresis

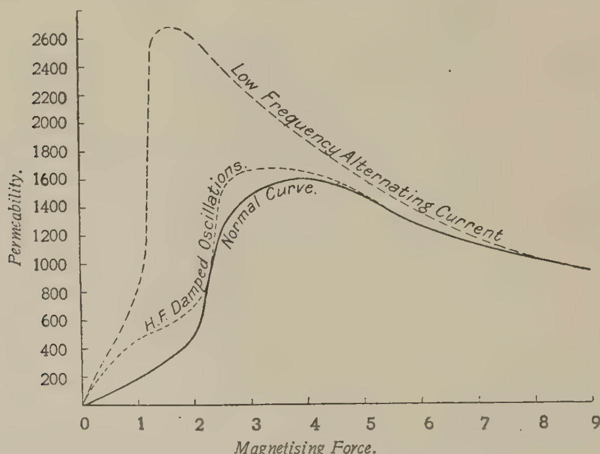


FIG. 20.

Effect of oscillatory fields on the permeability of iron wire.
L.F. current=60×10^{-3} amp. at 100 ∽. H.F. current=60×10^{-3} amp. at $0{\cdot}75 \times 10^{6}$ ∽ and decrement=0·68.

loss and to promote or increase the magnetisation at the extreme ends of the cycle. If, however, the cyclical force has a larger maximum (H=8 or more) then the general result is to diminish the hysteresis loss. Except in the case of strong currents (10 to 20 amperes) sent along the wire the general effect of either longitudinal or circular alternating magnetising force, damped or undamped, is to increase the magnetisation at the ends of the cycle; in other words, to increase the apparent permeability corresponding to the maximum magnetising forces. The oscillations or alternating currents, which, though taken alone, have relatively a small (R.M.S.) magnetising force,

appear to shake up the iron and so promote the magnetisation by the slowly periodic longitudinal magnetising forces.

Experiment shows that mechanical shocks or vibrations act very much in the same manner. If an iron wire is placed in a helix and a weak magnetising force $H < 1$ applied, then small twists or tapping the wire greatly increases the resultant magnetisation. If the magnetising force is removed and the twisting or mechanical shock subsequently applied then the remanent magnetisation is reduced.

In all cases it is found that the increase of hysteresis loss, and of apparent permeability at the extreme end of a cycle of magnetising force, reaches a maximum value for some particular strength of current through the wire. The value of this optimum current (R.M.S. value) depends upon the maximum value of the slowly cyclical force, being greater for small than for large maxima. It also depends upon the nature of the current whether high or low frequency, damped or undamped. In general it appears to be much greater for high-frequency damped oscillations. Hence it is probable that if very strong oscillatory currents were sent along an iron wire there would be a decrease in hysteresis under slowly periodic longitudinal magnetising forces of even small maximum value.

There is clearly a diminishing action on the hysteresis if the current along the wire is strong enough, but otherwise the action of the oscillations is to increase hysteresis. The joint effect is, therefore, the resultant of two opposing actions, one tending to increase and the other to diminish hysteresis.

It would occupy far too much space to make even a brief reference to the extensive previous work done on the subject of the effect of electric oscillations on the magnetic hysteresis and permeability of iron and steel. The investigations of Dr. W. H. Eccles, Dr. J. Russell, Mr. L. H. Walter, M. C. Mauvain, and also the contributions of Ascoli, Arno, Piola and P. Duhem on this subject are well known. We think it will be found that the above photographs taken with the campograph are not inconsistent with, but generally confirm, the conclusions of the above-named investigators. One thing is certain, viz., that the result of electric oscillations on iron superimposed on a slowly cyclical magnetic force is not always to diminish or to increase the hysteresis loss, but the results are dependent upon a number of factors and cannot be embodied in any short statement. The effects greatly depend upon the relative value of the maximum magnetising forces which are imposed on the

ron, one the slowly cyclical, and the other the rapidly varying one.

A result of considerable interest and perhaps of practical value is the proof given above that the passage of a strong unvarying or else alternating current through an iron wire immensely decreases the hysteresis due to a superimposed slowly varying longitudinal magnetic force, and that in some cases this may result also in a very considerable increase in the apparent permeability of the iron to this longitudinal force. The above results carry with them an explanation of the action of magnetic detectors of all kinds in wireless telegraphy, but it would occupy too much space to enter into the discussion of them.

ABSTRACT.

At a meeting of the Society in March, 1915, Dr. J. A. Fleming exhibited an instrument which he called a campograph, for photographing and delineating physical curves. Since then the optical arrangements have been greatly improved, and the point of light travelling over the photographic plate is now extremely small and sharp. With this improved campograph the authors have investigated the effect of electric oscillations on the hysteresis and permeability of iron. The general results are as follows :—

1. When an iron wire is taken slowly through a magnetic cycle and a superposed high-frequency magnetising force is also applied, then, if the maximum value of the slowly periodic longitudinal magnetic force does not exceed a certain value, the effect of the oscillations is to increase the area of the hysteresis loop and to increase the magnetisation at the ends of the cycle.

2. If the slowly periodic force has a large maximum value then the hysteresis loop is diminished in area, but the maximum magnetisation remains unaltered.

3. The increase in area of the hysteresis loop is generally less for high frequency than for low frequency oscillations, because in the latter case the oscillatory flux penetrates further into the iron wire.

4. If oscillatory currents are passed along the iron wire whilst at the same time the iron is taken slowly through a longitudinal magnetic cycle with continuous current, then when the oscillatory current has a small R.M.S. value the effects are generally similar to those produced by longitudinal oscillatory magnetic forces. If the oscillatory current is relatively large then their effect is to reduce the hysteresis and magnetisation at the ends of the cycle. This last action is due to the circular magnetisation produced by the longitudinal current, which grips the magnetic molecules of the iron, and prevents their longitudinal colineation and reduces also the hysteresis.

The same effect is seen when a longitudinal continuous current is passed along an iron wire.

5. The effect of superimposing on a steady feeble longitudinal magnetic force an alternating magnetic force either by undamped or

damped oscillations is greatly to increase the permeability at the ends of the cycle, provided the oscillatory force does not exceed a certain value. Beyond that a diminution sets in.

Photographs of the various effects were shown.

DISCUSSION.

Prof. Howe thought the Paper was a valuable addition to the knowledge of the effects of oscillatory fields on the magnetisation of iron.

Mr. D. Owen thought the rapidity with which investigations could be made with the instrument was very helpful. One would expect that the effect of a strong current passed along an iron wire which was magnetised longitudinally would be to remove the longitudinal magnetisation completely. This, however, was not the case. In some experiments he had once performed on the effect of currents along the wire superposed on a longitudinally magnetising force it appeared to be immaterial whether the currents were alternating or direct.

Lieut.-Col. Squier admired the instrument. The results would, he thought, repay careful study.

Dr. C. Chree was reminded by many of the results described of the combined effects of mechanical stresses and magnetic forces. Mechanical oscillations produce many similar effects, and he thought it possible that some of the observed phenomena were due to such mechanical oscillations being set up by the alternating magnetic forces.

Dr. S. W. J. Smith also thought that the results were mainly due to mechanical oscillations set up by the varying fields. Considerable work had been done on the subject. He quoted several observers who had described very similar phenomena produced by mechanical treatment, and indicated how oscillation of the molecular magnets gave rise to several of the effects.

Prof. Fleming, replying for the authors, said that it had only been possible to give a very rapid sketch of the phenomena. Many observers had worked on the subject, and he did not think any of the results described in the Paper were antagonistic to those already accepted. The principal point of the present Paper was, however, to indicate the rapidity and ease with which such experiments could be made by means of the campograph.

V. *A Hydraulic Analogy of the Wheatstone Bridge, by* Mr. R. S. Whipple.

Demonstration at the Meeting of November 12, 1915.

The analogy of the flow of water along pipes is frequently used when teaching Ohm's law, but the author does not know of a published description of a simple model. About 18 years ago Prof. H. L. Callendar showed a model of a pneumatic Wheatstone bridge in which the arms of the bridge consisted of small capillary tubes, through which a stream of gas or air could be passed. The galvanometer, or differential pressure indicator, as it was in this case, consisted of a mica vane pivoted in such a manner that it was deflected whenever the pneumatic pressure became greater on one side of the vane than on the other. By means of taps and tubes of various lengths, Prof. Callendar was able to show the effect of increasing or diminishing the length of pipe through which the gas passed.

The objection to the pneumatic model from the teacher's point of view is that the pressure indicator is difficult to make, and that all the taps require good workmanship. In the model shown, glass and indiarubber tubing with some glass taps are all that is required. The solution used is water coloured with ink, and this is allowed to flow slowly through the bridge circuit.

The galvanometer or detector is simply a large air bubble in a horizontal glass tube placed across the bridge arms. A tap is placed in the tube for adjusting the size of the bubble. When the capillary resistance of the tubing on each side of the bridge is equal, the bubble is stationary in the centre of the tube. As soon as the balance of the bridge is disturbed by either partially closing a tap or by introducing another length of tubing by the opening of a tap, the want of balance is shown immediately by the movement of the air bubble. Balance is restored by introducing or diminishing the resistance in the opposite arm of the bridge, and the bubble at once takes up its null position.

VI. *Experiments with Filaments Heated Electrically in Volatile Liquids.* *By* S. W. J. SMITH, *Hon. Sec. Physical Society.*

RECEIVED SEPTEMBER 24, 1915. READ MAY 22, 1914.

1. I was led to perform these experiments by a statement made to me by Mr. C. W. S. Crawley* to the effect that, when a 100 volt lamp filled with paraffin oil is used as a resistance in a 200 volt circuit, the bubbles of gas which form on the filament, instead of rising at once, run down the legs of the filament before they escape to the surface of the liquid.

Attempting to reproduce this phenomenon, I found that if the voltage applied to the lamp were reduced below 100, so that no bubbles formed, it was possible (by momentarily increasing the voltage) to obtain a single bubble upon the wire.

The behaviour of this bubble was most fascinating to watch, and seemed even more mysterious than the observation to which my attention had been drawn.

The bubble ran backwards and forwards from one terminal to the other, " looping the loops " of the filament many times before escaping from the wire !

2. Many other liquids, besides paraffin oil, yield the same result. Benzene, turpentine and aniline may be mentioned as examples.

The currents necessary for the production of the bubble and for its maintenance are higher, the higher the boiling point of the liquid. This suggests that the bubble consists of vapour of the liquid, and the inference is, in fact, usually correct except

* NOTE BY MR. CRAWLEY.—" The effect in question was shown to me by Mr. Addenbrooke, who was, however, too much occupied to bring it forward himself. The ' pip ' of an ordinary incandescent lamp had been broken off and the bulb filled with paraffin oil, for use as a resistance. It was found most satisfactory both as being able to absorb more power than with a vacuum and also to stand momentary overload much better, and for this reason alone it is worth mentioning.

" When the current is flowing there is naturally a strong current of hot oil up the legs of the filament. When sufficient current is used bubbles form on the filament, but these, instead of rising to the surface, run down the legs against both gravity and the upward current of hot oil and come off at the bottom. Having no facilities for investigating the phenomenon, I thought it might be worth while to draw attention to it in the hope that some member of the Society might be tempted to study it. Meanwhile, Dr. Smith has kindly looked into it and carried the matter further and, it seems to me, given the true explanation."—May 27, 1914.

during the first heatings soon after the liquid is poured into the lamp. Then there will be a certain amount of entrapped air which may provide the nuclei of the earlier bubbles. Indeed, if a very viscous liquid of high boiling point (such as machine oil) is used it is possible by means of a fine pipette to project a minute bubble of air upon the heated wire. This bubble then moves in the same manner as those formed spontaneously.*

3. Under favourable circumstances, with turpentine for example, it is easy to adjust the current so that a fairly large bubble circulates to and fro for a long time (a quarter of an hour, for instance) without appreciable change in size. Reducing the current a similar steady state can be reached with a bubble of reduced size. If the current be reduced below a certain limit, however, the bubble disappears.

These facts suggest that the contents of the bubble are continually changing, the constant size under any particular gradient being the result of a balance between the increase due to evaporation into the bubble and decrease due to condensation from it.

4. For, assuming as a rough approximation that the bubble is spherical, consider the effect of supposing its temperature to be uniform. The excess pressure within it will be given by $pr=2\sigma$, where r is the radius and σ is the surface tension, at temperature θ, of the bubble. Since p falls and σ rises when the temperature is lowered, it follows that the equilibrium value of r would be increased by lowering θ. But the bubble actually becomes smaller when the temperature is reduced. We must, therefore, conclude that its temperature is not uniform. The gradient from the wire must be such that, although the hottest part of the smaller bubble is colder than that of the larger one, its mean temperature is above that of the larger one.

For a restricted range it is possible that when the temperature of the wire is reduced the mean temperature of the diminishing bubble may rise ; but so soon as this becomes impossible the bubble must collapse, as experience shows.

It will also be noted that a spherical bubble of uniform temperature would be unstable under the conditions of the experiment.

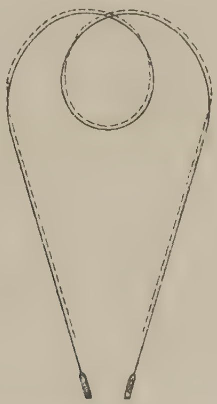
To sum up, it appears from a variety of considerations that

* I have to thank my colleague, Mr. H. Moss, M.Sc., for kindly undertaking to perform this experiment.

the bubble exists in a region of which the temperature changes rapidly in the direction perpendicular to the wire.

5. At first sight it seems possible that the movements of the bubble are due to electrical forces ; but, except that these may play some part in causing the bubble to cling to the wire, it is unlikely that they are appreciable.

The motion of the bubble is not affected by altering the direction of the current. It is produced equally well by alternating currents. It is thus apparently of thermal origin. and seems to be determined only by the temperatures to which the current is capable of raising the wire and its environment.



For convenience, the attempt to explain what is observed may be divided into two parts. Thus we may inquire why a moving bubble should follow the curvature of the wire before attempting to fix the cause of its motion.

6. A reason why a small rising bubble should follow the bend of the filament is not far to seek.

Keeping the current as low as possible, so that the motion of the bubble is comparatively slow and the convection currents from the heated wire are not very pronounced, it is possible to observe fairly easily the track which the bubble follows.* This track is represented by the dotted curves in the figure, the continuous curves denoting the filament of the lamp. The bubble may be said to pass along the "upper" side of each element of the filament. In other words, it runs along the

* Observation is facilitated by substituting a rectangular vessel, with plane glass sides, for the bulb of the lamp.

region in which the convection of heated liquid from the filament is a minimum. It is easy to see how such a region arises—permitting a layer of relatively hot liquid to remain in contact with the wire. This "sheltered" layer will be hottest and therefore least viscous nearest the wire. The rate of increase of viscosity outwards will be greatest in a liquid for which the conductivity is small and the temperature coefficient of viscosity large. The bubble, moving along the wire, will be moving in a region in which the viscous resistance to the motion of that half of it which is nearer the wire is less than that experienced by the more distant half. Hence the bubble will be subject to a force tending to press it towards the wire, and this force will be greater the greater its velocity.

7. The above consideration supplies a partial explanation of the phenomenon, for it shows why a bubble may proceed for some distance round the bend of the filament with the impetus acquired during its rise.

Indeed, the necessity for a sufficient gradient of temperature near the wire, if the bubble is to follow it, can be shown in a very simple way.

With a liquid of low boiling point, such as ether, for example, it is only possible to obtain indications of the phenomenon under ordinary circumstances. Thus it is possible to obtain bubbles which will rise along the filament to its summit, and perhaps go a little way beyond, but it is impossible to make them descend very far. The buoyancy exceeds the force pressing the bubble towards the wire, and it escapes to the surface of the liquid.

But if a steeper temperature gradient from the wire to the liquid is made possible, by cooling the ether in a freezing mixture beforehand, it is easy to obtain bubbles which descend as in other liquids of higher boiling points.

Conversely if we take a liquid which shows the phenomenon under ordinary circumstances, we can cause its disappearance by raising the temperature of the liquid, sufficiently, beforehand.

8. Assuming that the variation of viscosity which accompanies the temperature gradient operates in the way indicated above, it remains to consider the origin of the moving force, other than gravity, which acts upon the bubble.

This force must obviously have a considerable component parallel to the wire, and this must act, at every instant, in the direction in which the bubble is moving.

That this moving force depends upon the current strength, and, therefore, upon the temperature gradient, is easily shown. Thus, if the current strength be slowly diminished, after the bubble has formed, the oscillations of the latter gradually diminish in amplitude until the motion extends over only a small arc near the summit of one of the limbs of the filament.

The moving force is now so small that a very small fraction, acting parallel to the wire, of the opposing force due to the buoyancy of the bubble is sufficient to bring it to rest.

9. In attempting to explain how the moving force arises, suppose, for simplicity, that the bubble is momentarily at rest upon a horizontal wire. It will be in a region where the successive filaments of liquid, above and parallel to the wire, are of decreasing temperature. These filaments will be in unstable equilibrium, the upper ones being heavier than the lower. The latter would rise through the former but for the presence of the wire.

Now imagine a small displacement of the bubble along the wire. This will tend to elevate the liquid immediately in front of the bubble and to depress that immediately behind it. To convey roughly what is meant, the case of a sphere moving along the bottom of a horizontal trough containing a layer of water, with a layer of oil above it, may be cited. If the trough has glass sides, the water can be seen to rise in front of the sphere whilst the oil falls in behind the sphere as it moves. This takes place although the oil is much less dense than the water and the surface tension between them is considerable. A similar but more pronounced effect may, therefore, be anticipated in the case under consideration where the upper layers are denser than the lower and surface tension is inappreciable.

It is conceivable that this action once begun might continue of itself and keep the bubble moving forward. But, apart from this possibility, the colder liquid falling in behind the bubble will at once expand owing to its proximity to the wire and will thus tend to push the bubble forward in the direction in which it has begun to move.

The same kind of impulse will be produced in a similar way when the bubble is displaced downwards on a wire which is not horizontal and may be sufficient to enable it to move "against gravity."

In accordance with the facts, the distance through which the bubble is able to move in this way will depend upon the steepness of the temperature gradient, *i.e.*, upon the current flowing through the lamp.

The foot of each limb of the filament is comparatively cold owing to its nearness to the thick leading-in wire. Consequently the downward impulse will rapidly diminish as the bubble nears the end of the wire. The upward influence of gravity* will thus eventually assume predominance, bringing the bubble to rest and then starting it upon its return path.

10. According to the views just expressed, the bubble is continuously removing hotter layers of liquid from the neighbourhood of the wire and replacing them by colder ones. Consequently, in order to dissipate electric energy at a given rate, the temperature of the wire need not be so high when the bubble is present as when it is absent.

To find whether the difference was easily perceptible some observations were made with a platinum filament immersed in turpentine.† A bubble having been formed upon the wire, simultaneous readings of the current through it and the voltage between the terminals were made, as the current was reduced, with the following results :—

Bubble on wire.	
Amperes.	Watts.
2·5	4·25
2·4	3·87
2·3	3·49
2·2	3·18

The bubble disappeared when the current was lowered to 2·2 amperes. The current was then gradually raised to 2·5 amperes again (there being now no bubble present) and simultaneous readings of current and voltage were taken as before :

No bubble.	
Amperes.	Watts.
2·5	4·34
2·4	3·93
2·3	3·55
2·2	3·18

* In my experiments the upward currents of hot oil referred to by Mr. Crawley were inconspicuous, except near the vertical parts of the filament.

† A large quantity of turpentine was used in order to reduce, as much as possible, fluctuations in the temperature of the liquid as a whole during the course of these observations. Check measurements were made to ensure, for example, that the differences shown in the Tables were not attributable solely to a gradual rise in temperature of the liquid during the observations. From their nature the data are not to be regarded as more than qualitatively correct. They may, moreover, indicate merely the effects of general disturbance (equivalent to stirring) produced by the moving bubble.

From these data it will be seen that, when equal quantities of energy are being dissipated, the current is always greater, and therefore the resistance is always less, when the bubble is present than when it is absent. Since the wire is of platinum, the temperature must be less when the resistance is less.

If the bubble is watched it is seen to be followed by a trail of liquid, evidently of very different density from the main bulk, moving outwards from the part of the bubble furthest from the wire. Such a trail would be formed by hot liquid thrown off by the moving bubble (see also below).

11. It has been inferred (§ 4) that a bubble of vapour cannot remain of constant size except under continuous evaporation and condensation of liquid. This can be demonstrated in the following way :—

When the amplitude of the oscillations of the bubble is continuously reduced in the way described in §8, the bubble frequently becomes so small that it comes to rest before it disappears, by adhering to a minute thread or roughness (visible through a microscope) upon the wire. It is generally possible to adjust the current so that this stationary bubble remains of constant size instead of gradually disappearing. It is then easy to observe that a stream of hot liquid is rising from the side of the bubble remote from the wire.

It is probable that in this case the film of liquid between the bubble and the wire is excessively thin. The central part of the film may even disappear. Liquid will be approaching it continuously owing to capillarity, but may be volatilising at such a rate as to keep the film interrupted at its centre. The wire will tend to become very hot at this point, for it must be remembered that the current through the wire (when a bubble is present) is always such that the wire would become incandescent if the liquid were removed. The evaporation into the bubble near the wire is compensated by an equal condensation at the remote part of the bubble, the temperature being kept below the boiling point here by the mixing of the condensed vapour with the surrounding cooler liquid. This process produces the rising stream.

It will be noticed that the displacement of such a bubble in either direction along the wire would bring fresh liquid over a very hot part of the wire. The sudden heating and expansion of this liquid would provide an impetus tending to move the bubble very rapidly in the direction of displacement.

12. An exhaustive consideration of all possible causes of

motion of the bubble has not, of course, been attempted. It is possible that the explanations put forward above, which are reminiscent of those given in the case of the Trevelyan Rocker, are incomplete. For instance, differences of surface tension due to difference of temperature between the two sides of the bubble may produce an effect which has been disregarded. And it may easily happen that a better way of explaining the phenomena will occur to some observer tempted, by their interesting and possibly instructive nature, to take the small amount of trouble required to produce them.

ADDED OCT. 21, 1915.

Since writing the above account of experiments performed in May of last year,* it has occurred to me that it might be instructive, in experiments like those of §10, to obtain some idea of the temperature of the wire in the different cases. Some additional observations, with a wire similar to that previously used, have therefore been made. The results are tabulated below :—

Amperes.	Volts.	Resistance.	Temperature.
2·85	2·20 (no bubble)	0·772	104·0°C.
	2·17 (bubble)	0·761	99·0°
2·95	2·31 (no bubble)	0·783	109·5°
	2·26 (bubble)	0·766	101·5°

The numbers in the first and second columns are currents and corresponding voltages (mean values) obtained as in §10. The numbers in the third column are the resistances obtained by dividing the volts by the amperes in each case.

To obtain the numbers in the fourth column, the liquid was heated by an external source to about 130°C. It was then allowed to cool slowly, being kept stirred meanwhile. The resistance and temperature of the wire were measured from time to time. The numbers in the fourth column give the temperatures at which the wire had the resistances shown in the third.

From these data it is seen that, both with and without the bubble, the average temperature of the wire is 50°C., or more below the boiling point of the turpentine (approx. 160°C.). The temperature of the bubble itself cannot be below the boiling point. It is thus always much hotter than the liquid which it

* *See* "Bulletin," Phys. Soc., 1914, p. 44.

displaces during its motion. But its temperature is maintained by the wire. Therefore that part of the wire upon which the bubble bears at any instant is much hotter than the rest. It is easy to see how this may occur. The thin film sandwiched between the hot bubble and the wire, no longer able to transfer the heat supplied by the wire to surrounding colder layers, is practically instantaneously raised to the boiling point.* The bubble is maintained by evaporation from it.

These experiments show, perhaps more explicitly than any of the others, where the force propelling the bubble originates.

ABSTRACT.

The first of these experiments was due to Mr. Addenbrooke who, using a 100-volt lamp filled with paraffin oil (after removing the tip) as a convenient high resistance in a 200-volt circuit, noticed that some of the many bubbles forming on the filament behaved in a curious way. Instead of rising at once to the surface from the point at which they formed they ran down the legs of the filament, against gravity, and then escaped at the leading-in wires.

Dr. Smith, led to repeat this experiment by Mr. Crawley, discovered another, more striking, phenomenon. Placing the 100-volt lamp in a 100-volt circuit in series with a variable resistance (conveniently a water-trough) it was found possible, by momentarily cutting out most of the resistance, to obtain a single bubble upon the wire. The behaviour of such a bubble is very interesting to watch. Instead of escaping at either terminal, as in Mr. Addenbrooke's experiment, it travels backwards and forwards between the two, "looping the loops" of the filament in a fascinating way during every journey.

The peculiarities of this phenomenon, which can be obtained with either direct or alternating supply, have been analysed by examining the size and motion of the bubble under various conditions and also by using filaments of different materials and liquids of different boiling points.

It was shown, from the experiments, that a rapid fall of temperature from the wire through the liquid, in the region through which the bubble moves, is an essential condition of the phenomenon, and also, from theoretical considerations, how this condition can be used to explain why the bubble moves in the manner described.

* In the experiment by Mr. Moss, referred to in § 2, some of the air bubbles being too large did not attach themselves to the wire as intended. Passing across the wire, they rose to the surface of the oil and, bursting there, discharged minute clouds of smoke. The short time during which a film existed upon the wire, while the bubble passed across it, had sufficed to enable the current to heat the film to decomposition.

VII.—*On Obtaining and Maintaining a Bright Hydrogen Spectrum with Special Reference to the* 4,341 *Line.* By

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Synopsis.

1. *Introduction.*

In the system at present adopted for the classification of optical glasses, use is made of the following spectrum lines in terms of which to specify their refractive and dispersive properties :—

Spectrum line.	Wave-length.	Source.
K_α or A′	7,682 Å.U.	Potassium flame.
H_α ,, C	6,563 ,,	Hydrogen vacuum tube.
D_1	5,896 ,,	Sodium flame.
H_β or F	4,861 ,,	Hydrogen vacuum tube.
H_γ ,, G′	4,341 ,,	Hydrogen vacuum tube.

For all ordinary purposes the A′ line is superfluous, and is rarely included in determinations of refractive index. The practical standards for routine work are therefore the sodium line and the C, F and G′ hydrogen lines.

This system has been much criticised,* principally on account of the unsuitability of these lines for accurate work. Of these the G′ is by far the least satisfactory, and has probably been the bane of every observer who has used it in practical optics. Vacuum tube discharges are of small intrinsic bright-

* See, for example, Dr. Martin Lowry, "Proc." Phys. Soc., XXIV., VI., 400; and Lt.-Col. Gifford, "Optician," July 2, 1915; also discussion on same, "Optician," Aug. 27, 1915.

ness at best, and when used in conjunction with a Pulfrich refractometer, which is itself very wasteful of light compared with a spectroscope, it is essential to employ relatively heavy excitation to obtain lines bright enough to give the ease of setting and freedom from eye-strain essential to accurate work. With a current of 15 to 20 milliamperes in a new tube, or one which has been newly re-exhausted, plenty of light is obtained in all the required lines, but the tube deteriorates so rapidly that in a very short time the G′ is hardly visible while the F line has lost much of its initial brilliance. The author has frequently found the G′ of a new or freshly prepared tube disappear completely in the course of an afternoon's work, despite the most rigorous economy in the discharge and the time during which it was running. The chief difficulty, therefore, is not in obtaining sufficient brightness in the required lines but in maintaining it for any length of time, and the object of the present investigation was to determine the conditions governing the intensity of the lines, especially G′, with a view to mitigating as much as possible their rapid deterioration under the influence of the discharge.

2. *Previous Work.*

With regard to the relative brightness of the lines of the series spectrum of hydrogen some interesting results were described by Mr. H. L. P. Jolley* who measured the energy of the C, F, and G′ lines under certain conditions of discharge with a Rubens thermopile and Paschen galvanometer. Unfortunately, in order to obtain sufficient energy, Jolley was obliged to use heavily condensed discharges at pressures of several centimetres of mercury in water-cooled quartz tubes. He was therefore working under conditions quite outside the limits of ordinary practice, but it is of interest to note that under these conditions the energy becomes more concentrated at the red end of the spectrum as the discharge current or the pressure of the gas is increased. Jolley also measured the total energy of the uncondensed discharge and found it to be very nearly proportional to the discharge current over a considerable range. P. G. Nutting and Orin Tugman† showed that in the case of the uncondensed discharge in hydrogen the

* "Phil. Mag.," XXVI. 1913, 801.
† "Bulletin," Bureau of Standards, VII., 1911, p. 49.

energy is more concentrated at the red end of the series spectrum for high currents than for low. Thus, with 380 milliamperes, C was 60 times, F 24 times, and G′ only 12 times as bright as with a current of 20 milliamperes, the pressure being 1 mm. They found that the intensity of the lines at constant current was a maximum at about 1 mm., diminishing as the pressure was altered either above or below that value. There was no selective effect in this case corresponding to that observed by Jolley for condensed discharges.

G. Stead* showed that with hydrogen, in whatever way prepared, the series spectrum is bright at the cathode and faint at the anode, while the secondary spectrum is brightest at the anode. Sir J. J. Thomson† also found that the intensity of F relative to that of C was much less near the anode than the cathode.

B. Hasselberg‡, M. A. Dufour§ and others have found that the secondary spectrum is favoured by weak and the primary spectrum by strong discharges.

W. E. Curtis|| states that if hydrogen be thoroughly dried by standing over P_2O_5 for some time, the series spectrum is comparatively feeble. Curtis also experienced difficulty with the rapid deterioration of the series lines when the discharge current was high, a quarter of an hour being sufficient in some cases to necessitate refilling of the tube. To get over this, Curtis employed the elegant device of continuously sucking the gas through his discharge tube from a reservoir at atmospheric pressure, a length of very fine capillary being interposed to maintain the difference of pressure between that of the gas in the reservoir and that in the discharge tube. This appears to have given entire satisfaction for the duration of the longest exposures required.

P. G. Nutting¶ in a series of Papers dealing with the preponderance of one spectrum over another in a mixture of two spectra treats of the effects of capacity and inductance on the character of the discharge in various gases, including hydrogen. It should be noted that Nutting, following Plücker and Hittorf, uses the terms *primary* and *secondary* in the reverse sense to that of the other authors quoted. Thus the many lined

* "Proc.," Roy. Soc., A, 85, 1911, p. 393.
† "Proc.," Roy. Soc., 18 5, Vol. 58, p. 244.
‡ "Mem. de l'Acad," de St. Petersburg, VII., Vol. 31, p. 14.
§ "Ann. de Ch. et de Ph.," (3), 1906, Vol. 9, p. 361.
|| "Proc.," Roy. Soc., Vol. XC., 1914, p. 605.
¶ "Bulletin," Bureau of Standards (1), 1904-5, pp. 77, 83, 399.

spectrum which appears in the uncondensed discharge in hydrogen at high pressures is termed the *primary* spectrum by Nutting, whereas in those of the other Papers quoted above in which the terms *primary* or *secondary* occur this is referred to as the *secondary* spectrum. The latter practice is the one followed in the present Paper.

3. *Apparatus Employed in Present Work.*

All the vacuum tubes used were of the ordinary "H" type with spiral aluminium electrodes. They were, in fact, tubes which had been purchased at various times exhausted and ready for use and which had been laid aside when too unsatisfactory to employ further. The capillaries were about 4 cm. long and 1·5 mm. in diameter. They were used "end on."

To produce the discharge two induction coils were available, one of $2\frac{1}{2}$ in. and the other of 6 in. spark-length, the latter being used in conjunction with a rotary mercury break. This was much steadier and less noisy than the hammer break of the coil.

The Leyden jars used had each a capacity of 0·00075 mfd., while the inductance consisted of a single layer of No. 28 double silk covered copper wire wound on a waxed cardboard cylinder of 15 cm. diameter and 75 cm. in length. Connections from various points of the coil to a rotating dial enabled the inductance to be varied in steps up to about 75 millihenries.

Measurement of the discharge current was effected by passing it through a short straight length of constantin wire (1ω per centimetre) in series with the tube. About 2 mm. above and parallel to this wire a linear thermopile consisting of 20 iron-constantin elements in a length of 1 cm. was mounted. This registered on a Paul semi-pivoted microvoltmeter of 10ω resistance and full-scale reading 0·3 millivolt. This proved a very satisfactory arrangement for the range of current employed, the full deflection being obtained when 45 milliamperes passed through the heater, but it could only be used for uncondensed discharges. If a condenser was employed or if a spark gap was opened in series with the tube, it appeared to be impossible to prevent discharges passing between the heater and the thermopile which vitiated the readings completely. The author is indebted to Mr. A. Campbell for much experienced advice on the measurement of high potential currents.

4. *Features of Uncondensed Discharge.*

The general effects of varying the pressure in a hydrogen vacuum tube on the character of the spectrum are well known. At pressures of 5 cm. or 6 cm. of mercury the energy is almost wholly confined to the many-lined or secondary spectrum, while the lines of the Balmer series are faint and lustreless, G′, the most important from the point of view of this Paper, being scarcely visible. As the pressure diminishes the energy progressively passes from the secondary to the series spectrum, which gradually brightens up. It is not, however, until the pressure is under a centimetre that G′ can be described as good. At 5 or 6 mm. all the lines shine out with a brilliant lustre which continues to improve until about a millimetre is reached. If the pressure is reduced much below a millimetre* the resistance of the tube begins to increase, and the brightness of the lines is diminished.

It will be seen, therefore that the most favourable range of pressure for obtaining a satisfactory G′ line is from 1 mm. to 5 mm. of mercury.

Having exhausted the tube to a pressure within this range, the actual brightness of spectrum obtained will depend on the discharge current, the only limit to its increase being set by the breaking of the tube. Using a water-cooled tube and 6-in. coil, the author was able to obtain a G′ line quite as bright as the violet mercury line given by a particular water-cooled quartz arc taking 6 amperes.

The places where fractures are likely to occur are the capillary and the ends of the tube in the neighbourhood of the electrodes. In order to determine how the heating of these parts depended on the conditions under which the tubes were run, a single thermocouple of copper-German silver was tied firmly to the capillary with cotton thread, so that the junction was above and opposite the middle of the capillary. Suitably shunted, the couple was connected to the Paul micro-voltmeter already mentioned. The reading of the latter indicated on an arbitrary scale the heat developed in the capillary. A similar junction was bound to the wall of the tube near where the cathode enters the glass. By means of mercury cups it was

* The actual pressure at which the resistance of the tube begins to increase depends on the dimensions and on the magnitude of the current. It is supposed to mark the stage when the further extension of the cathode glow is hindered by the walls of the tube.

easy to take successive readings of the two couples, and of the current in the tube.

In Fig. 1 the indications of the two couples are shown as the pressure of the tube is varied. The voltage on the primary of the induction coil was maintained constant throughout.

The curves show that the heat developed in the capillary decreases as the pressure diminishes, falling off very rapidly at low pressures, while the heating of the glass near the

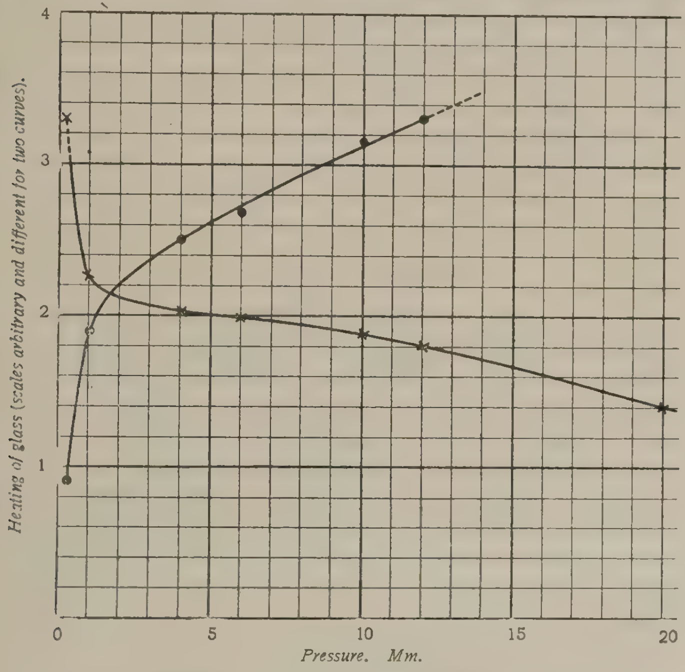


FIG. 1.—HEATING OF TUBE AT DIFFERENT PRESSURES.
(Constant voltage on primary of induction coil.)
● ● ● ● Capillary.
× × × × Glass near negative electrode.

electrodes is least at high pressures, rising gradually as the pressure falls. There is a rapid increase below 1 mm.

From these results it is clear that the danger of a cracked capillary increases rapidly as the pressure rises, while if the exhaustion is carried much below 1 mm. trouble with the glass near the electrodes may be expected. Hence at pressures of 1 to 3 mm. larger currents can be used with confidence than can safely be employed outside that range.

The author has found that most tubes of the ordinary commercial pattern will stand 30 to 35 milliamperes for a considerable time, while they will take as much as 45 milliamperes for short periods of 30 seconds to a minute, without serious risk, if rests of one or two minutes are allowed.

These currents are well within the capacity of a 6 in. coil. With a $2\frac{1}{2}$-in. coil such as is generally used for vacuum tube work in refractometry, &c., it has not been found possible to obtain more than 20 milliamperes. This required 5 or 6 amperes in the primary circuit, and gave continual trouble with the contact breaker. Nevertheless, while a coil of this size is not sufficient to get the best results that can be obtained from a hydrogen tube, it is quite sufficient, if the pressure is right, for most purposes for which the G′ line is required. It is quite sufficient for work with the Pulfrich refractometer.

5. *Effect of Capacity and Inductance.*

The effect of capacity on the hydrogen spectrum at different pressures is also well known. At high pressures (5 or 6 cm.) the series lines, which, in the uncondensed discharge are faint and dull, brighten up considerably, but are rendered completely diffuse. A short spark gap in series with the tube enhances the effect enormously, the secondary spectrum being almost destroyed. The diffuseness of the lines, unfortunately, renders them wholly useless for refractometry or any purpose for which sharp lines are essential. If, however, a suitable inductance be put in series with the tube and spark gap, the lines may be rendered perfectly sharp, and, though less bright than without the inductance, are still many times more so than when the uncondensed discharge is employed. Moreover, the secondary spectrum is still much enfeebled while any lines due to residual air are quenched.

As the pressure is reduced the enhancing effect of capacity and inductance decreases until at about 1 cm. it is negligible, although it is of interest to note that over a small range the brightness of C may be slightly diminished while that of G′ is appreciably increased. Below this the effect of capacity and inductance is to diminish the brightness of all three lines, though there is still an appreciable increase in the contrast between the lines and their surroundings, due to the partial suppression of the secondary spectrum which appears to take place to some extent even at the lower pressures.

In these experiments it was found that two Leyden jars were distinctly better than one, while three were probably still better, but no improvement was obtained on adding more capacity. Inductances of between 20 and 40 millihenries seemed equally useful.

The effect of inductance on the condensed discharge in hydrogen for a particular pressure is shown very clearly in a photograph illustrating one of the Papers by P. G. Nutting*, referred to in section 2. Unfortunately the pressure is not stated. With the largest inductance employed (about one-twentieth of that used by the author) the series lines are seen to be quite sharp while the background of secondary spectrum is relatively feeble.

The practical importance of these results will be shown in a subsequent section.

6. *Effects of Impurities.*

When reasonable precautions are taken there need be no gaseous impurities present in the tube in sufficient quantity to affect the brightness of the hydrogen lines. It is much more difficult, however, to prevent the presence of traces of mercury in the tube, especially if mercury pumps and pressure gauges are used Mercury is a double nuisance in a hydrogen tube. The slightest trace of it detracts enormously from the hydrogen spectrum† and the violet mercury line at $\lambda=4{,}359$ is so near G′ that in the field of a Pulfrich refractometer it is difficult to separate the two bands without reducing their brightness considerably. If they are not separated, the mercury band overlaps the critical edge of the hydrogen band and accurate setting is impossible. For these reasons it is essential to keep the tube perfectly free from mercury, and, while it is possible to do this using mercury pumps, it is much safer to use an oil pump and to dispense with a pressure gauge. It is easy to judge from the character of the discharge when the proper stage of exhaustion has been reached.

If a tube shows mercury lines it can be completely cleaned by washing out with strong nitric acid which should be run back and forward through the capillary and allowed to stand

* "Bulletin," Bureau of Standards (1), 1904-5, p. 83.

† P. Lewis, "Astroph. Journ.," 10, p. 137, 1899, has shown that if one molecule of mercury vapour be present for every three thousand molecules of hydrogen, the intensity of the hydrogen lines will be halved.

for some time in contact with the aluminium electrodes. After rinsing out with distilled water several times the tube should be dried in a hot air oven at about 250°C. The cleaning is facilitated if a temporary opening is made at *b*, Fig. 3. If this is not done it is difficult to get the liquids introduced to the portion *bc* or to get them back again.

7. *Deterioration of Hydrogen Tubes.*

When a new tube is run with a current sufficient to give a satisfactorily bright discharge, the character of the discharge rapidly alters. First of all the G′ fades. This is followed less

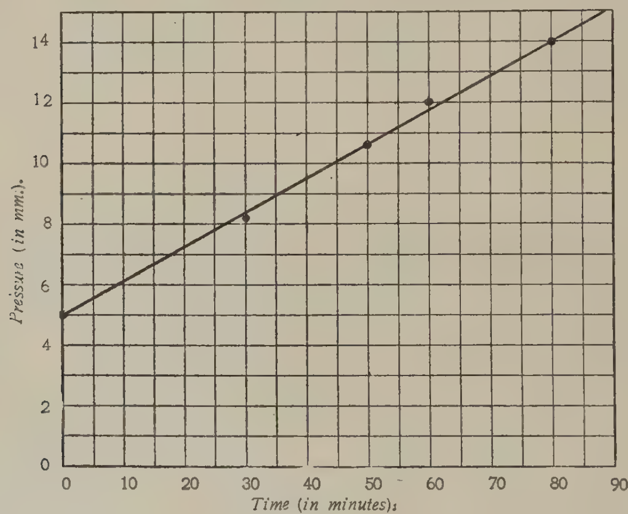


Fig. 2.

rapidly by the F line and eventually even C loses much of its initial brilliance. The background of secondary spectrum becomes relatively stronger, the positive column loses its red colour, while the sodium lines make their appearance. The tube, in fact, passes inversely through the various stages which may be observed if the pressure of the gas be gradually diminished. The deterioration is, therefore, due to a continual rise in pressure under the influence of the discharge. As an example of the rate of increase, Fig. 2 is of interest. The ordinates represent the pressure in a discharge tube through which a current of 35 milliamperes was running. The abscissæ give the time from the commencement of the test.

On stopping the discharge and allowing the tube to cool the pressure did not alter appreciably.

The increase, which is due to the evolution of hydrogen by the electrodes, is more pronounced in some tubes than in others, but is sufficient in all cases to render the ordinary type of discharge tube wholly useless for any purpose for which the G′ line is wanted, as the necessary current makes the frequency with which re-exhaustion is required extremely troublesome.

8. *Minimising the Deterioration.*

In section 5 it was shown that when the pressure in a discharge tube is under a centimetre of mercury the brightness of the series lines cannot be increased by using capacity and inductance, but that at higher pressures considerable improvement may be effected by this means. This at once suggests the use of capacity and inductance with tubes which have partially deteriorated. With a tube which was so far gone that no trace of the G′ line was visible in the field of a Pulfrich refractometer it was possible by introducing capacity and inductance to obtain a band sufficiently bright to make fairly good settings.

This method, while a very valuable reserve in case of emergency, is not, however, a satisfactory solution of the difficulty, inasmuch as it does not diminish the rise of pressure, which ultimately gets so high that the tube is unusable under any conditions.

Attempts to age tubes by running them with a heavy current for several hours before detaching them from the pump proved useless, as did also attempts to regulate the pressure, by means of charcoal or metallic absorbents. These presented too many difficulties to be practicable.

The attractive method employed by Curtis (*see* section 2) is obviously only feasible for purposes of research and much too cumbersome for use in routine work where the vacuum tube is only an accessory and has to be ready for use when required without any preliminary trouble.

The problem was eventually solved by attaching to the discharge tube a large auxiliary volume. This consisted of a spherical bulb* of about 6 in. diameter and was fitted to the

* The bulb can be obtained at trifling cost with neck already drawn out ready to attach at *a* from any manufacturer of X-ray apparatus.

discharge tube as in Fig. 3. This arrangement enables the capillary to be immersed in a glass water bath if it is desired to run the tube very heavily.

The presence of the auxiliary bulb diminishes the rate of change of pressure due to the evolution of hydrogen in the ratio of the volume of the discharge tube alone to that of the tube plus bulb, *i.e.*, by about 25 cubic cm. to 1,800 cubic cm., or 1:70 approximately. The useful life of the tube is thus prolonged by about 70 times, assuming that no diminution occurs in the rate at which gas is evolved. Any gradual diminution of this rate, were it to occur, would still further prolong the life of the

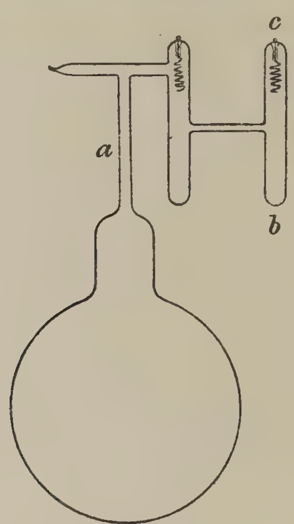


Fig. 3.

tube. The frequency with which re-exhaustion becomes necessary, even with heavy usage, is thus reduced to within reasonable limits. As a matter of fact, none of the tubes which the author has prepared have yet required refilling, nor do they show any serious evidence of deterioration. One of these, A_5, has been used for practically all the optical glass testing done at the Laboratory in the last four months, while another, A_6, has been used for a large amount of research work sometimes involving continuous action under discharge currents of 20 to 25 milliamperes for several hours on end. As a practical test of the new bulb-tube an ordinary tube, A_7, with no bulb, was exhausted to the same degree as A_6. A discharge of

20 milliamperes was then run through the tubes in series. At the commencement the discharges were exactly similar, but, at the end of two hours, the intensity of G′ in the spectrum of A_7 had fallen to 0·2 of its initial value and that of F to about 0·4. No change whatever was detected in the case of A_6.

9. *General Usefulness of Bulb-Tube.*

As indicated in the introduction, the work which has been described was rendered necessary by the exigencies of the refractometry of optical glass. The difficulty of obtaining a satisfactory G′ line from ordinary tubes involved considerable loss of time spent in taking a multiplicity of observations, while the strain on the eyes of the observer was a serious matter if much work had to be done at a time. Since the new type of tube was introduced, however, there has never been any difficulty in obtaining a suitable G′ line, and observations at this wave-length are now as easy and accurate as those on any of the other standard lines.

The usefulness of the bulb-tube is not, of course, confined to refractometry. For spectroscopic and spectrographic work the advantages of being able to maintain a vacuum tube discharge of constant character, even under heavy excitation, for considerable periods without external regulation are obvious. If a 6 in. coil is used sufficient G′ light can be obtained from a hydrogen tube for polarimetric and spectrophotometric work. Except in cases where the nature of the gas renders the use of small quantities imperative, the attachment of an auxiliary volume to dilute down changes due to varying pressure or to the development of impurities seems to be an advantageous proceeding in all vacuum tube work.

ABSTRACT.

The Paper treats of the conditions of pressure, discharge, &c., most suitable for the production of a bright hydrogen spectrum, such as is required for refractometry and similar purposes. The rapid deterioration of the tubes with use is shown to be caused by a rise of pressure due to the evolution of hydrogen by the electrodes. The trouble may be obviated by sealing an auxiliary bulb of 1½ or 2 litres capacity to the discharge tube. This reduces the rate of pressure variation and prolongs the useful life of the tube nearly a hundredfold. The use of capacity and inductance is shown to be very helpful with partially deteriorated tubes.

DISCUSSION.

Mr. F. E. Smith asked if the hydrogen could not be expelled from the electrodes before sealing off.

Mr. S. D. Chalmers said he had had a similar experience with the deterioration of hydrogen tubes. He was surprised that the author found all the lines to be most satisfactory at the same pressure. He had sometimes found it convenient to use different tubes for the C and G′ lines.

Mr. G. H. Gardiner mentioned that when taking some photographs of vacuum tube spectra he had been struck with the rapid deterioration of some of the ultra-violet lines. This had been traced to the formation of a scarcely perceptible deposit on the quartz window due to the scattering of the electrodes, and he thought a similar cause might be sufficient to account for the decay of the violet hydrogen line.

Prof. J. W. Nicholson thought the Paper, although written from another point of view, formed a useful contribution to the knowledge of the conditions under which different spectra of the same gas were produced. The hydrogen spectrum was a difficult one to deal with, and anything which threw additional light on it was welcome.

Mr. D. Owen referred to the rapid rise in the heating of the glass near the electrodes at pressures under 1 mm. What happened at still lower pressures ? Presumably the rise could not go on indefinitely, as, under the conditions of the experiment, the current decreased as the exhaustion progressed.

The Author, in reply, said he had tried to get rid of the hydrogen occluded in the electrodes before detaching the tube from the pump but without success. His experience that all the lines attained their maximum brightness at the same pressure was borne out by all other observers who had made quantitative observations. A tube which does not show any G′ may still have plenty of C and F on account both of the greater energy of these lines and of their greater visibility, but he could not conceive of a tube having a usable G′ being deficient in either C or F. It was shown conclusively in the Paper that the deterioration was due to pressure changes. No " imperceptible " deposit could affect lines in the visible spectrum, however effective it might be in the case of those in the ultra-violet. Moreover, no deterioration due to deposits on the glass or to the condition of the capillary would be retarded in the least by the addition of an auxiliary volume. With regard to the heating of the glass, the general form of the curves was easily explained in terms of well-known vacuum tube phenomena. The experiments, however, were intended to give practical rather than theoretical information, and the care expended on them was not such as to justify any conclusions being drawn as to what might happen at pressures below those actually reached.

VIII. *Determination of the Coefficient of Diffusion of Potassium Chloride by an Analytical Method.* By A. GRIFFITHS, J. M. DICKSON *and* C. H. GRIFFITHS.

RECEIVED JUNE 21, 1915.

§ 1. *Introduction.*

THIS Paper represents an attempt to develop an analytical method of determining the coefficient of diffusion of a salt in water capable of giving consistent and accurate results.

The lower ends of a number of vertical and parallel diffusion tubes end in a reservoir of large capacity containing a solution of potassium chloride. The greater part of the reservoir is above the lower ends of the tubes, and by gravity the solution at the lower ends is kept at an approximately constant concentration. The upper ends of the tubes are covered with a cap provided with an outlet and an inlet tube. Water enters the cap by the inlet tube, and a weak solution containing the diffused salt leaves the cap by the outlet tube. Time, which may be as long as a fortnight, is allowed for the attainment of the steady state, and an individual experiment may last six weeks. The quantity diffused is obtained by chemical analysis.

§ 2. *Theory.*

Let n_0=the concentration at the top of the tubes.
N=the contentration at the bottom of the tubes.
D=the density at the bottom of the tubes.
L=the length of each tube.
k=the coefficient of diffusion.
c=quantity of salt transmitted per unit area.
ρ=density of solution.
n=the concentration of the solution.

Let it be assumed that

$$\rho=1+an,$$

and let

$$b=1-a$$

V=downward velocity of liquid at bottom of tubes.

It can be proved that

$$-\frac{b(N-n_0)}{(v-bc)}+\frac{v}{(v-bc)^2}\log_e\frac{(v-bc)N+c}{(v-bc)n_0+c}=\frac{L}{k}, \quad . \quad . \quad (1)$$

where

$$v=V(1-bN)=V(D-N). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Under the conditions of the experiments of this Paper, by expanding the left-hand side of equation (1) it can be proved that the relation between c and k is represented approximately by

$$\frac{(\mathrm{N}-n_0)}{c}\left\{1-\frac{v(\mathrm{N}-n_0)}{2c}\right\}=\frac{\mathrm{L}}{k}. \quad \ldots \quad (3)$$

Let δ=the ratio of the increment in volume produced to the increment in the mass of salt dissolved for a solution of the strength of that of the reservoir; *i.e.*, it is numerically measured by the increase in volume produced when 1 gramme of salt is dissolved in a solution of the given concentration, the amount of the solution being so great that the addition of the salt makes no appreciable change in the concentration.

Obviously*

$$\mathrm{V}=c\delta \quad \ldots \quad (4)$$

From (2) and (4)

$$v=c(\mathrm{D}-\mathrm{N})\delta \quad \ldots \quad (5)$$

By substitution of this value of v in (3)

$$k=\frac{\mathrm{L}c}{(\mathrm{N}-n_0)}\times\frac{1}{1-\frac{1}{2}(\mathrm{D}-\mathrm{N})(\mathrm{N}-n_0)\delta} \quad \ldots \quad (6)$$

The term $\frac{1}{2}(\mathrm{D}-\mathrm{N})(\mathrm{N}-n_0)\delta$ has a value small compared with unity, and it is, therefore, unnecessary to determine the value of δ with a high degree of accuracy.

§ 3. *Apparatus and Method.*

A diagrammatic sketch of the final form of the apparatus is shown in Fig. 1. The diffusometer D is placed in a large accumulator tank filled with water. This accumulator tank was fixed in a strong wooden box, the space between it and the box being packed with cork dust. The box was covered with a lid, not shown in the diagram. The whole apparatus was placed in a room kept at a fairly constant temperature by means of an automatically controlled gas stove.

At one time vibration was suspected as a disturbing factor, and to minimise vibration in later experiments the box was suspended by a stout spring, S, and a vane attached to the

* It may be mentioned that

$$\delta=\left(1-\frac{d\rho}{dn}\right)\Big/\left(\rho-n\frac{d\rho}{dn}\right)$$
$$=[(\rho_1-n_1)-(\rho_2-n_2)]/(n_2\rho_1-n_1\rho_2) \text{ approximately,}$$

when n_1 and ρ_1 and n_2 and ρ_2 are neighbouring pairs of values of ρ and n.

bottom of the box dipped into a viscous liquid. In addition, the diffusometer D was suspended by a weak spring, *s*, from an upright fitted to the box.

Two methods were employed to obtain the slow flow of water. The first method gave a gradually decreasing rate of flow, and was ultimately discarded ; the second is suggested in the sketch. A tube, O_2, connected with O_1, the outlet to the diffusometer, passed freely through a hole in the stopper of the tube T, and dipped into liquid kept at a constant level by

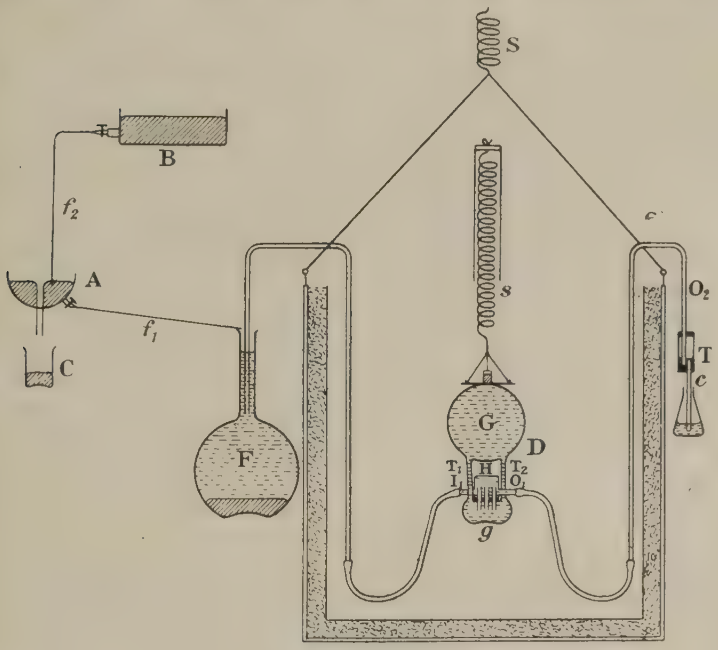


FIG. 1.

means of the overflow tube *c*. The supply reservoir was a large flask, F. To the water was added a trace of copper nitrate to keep down fungoid growths. As all the tubes connected with the diffusometer were wide, it may be assumed that, as a rule, the level of the water in the neck of the flask and that in T were approximately the same. Water was driven at a steady rate from the flask to T by allowing to fall into the flask from a fine capillary tube, f_1, a steady stream of small drops of mercury. The fine capillary tube was attached to a reservoir,

A, the mercury in which was kept at a constant level by means of an overflow tube. The supply of mercury was sustained by means of a second reservoir, B, placed at a higher level and giving through the fine capillary f_2 a slightly faster stream of mercury than that from f_1. Cleanliness of mercury from the beginning to the end is necessary for the success of the method.

It may be added that this method of giving a constant flow is self-adjusting so far as the water is concerned. If an obstruction should form, an automatic difference of level will be created and the flow will again become steady.

The general construction of the diffusometer D is suggested in the figure. G and g are two hollow glass bulbs connected by tubes T_1 and T_2. G is large compared with g. The upper part of g possesses a neck on which a diaphragm is inserted; through the diaphragm pass the diffusion tubes (or tube). H is a compartment provided with an inlet tube, I_1, and an outlet tube, O_1.

At the top of the apparatus was a small circular spirit level, and the three wires supporting the apparatus were so attached to screws that the apparatus could be leveled after it had been suspended in the tank.

The solution that came through the outlet tube from T was collected in a weighed flask, and after a measured interval of time was removed and chemically analysed. Usually two, and frequently three, analyses were made of each flask of solution, and the average of the results obtained was used in the calculation of K.

§ 4. *Details of Three Diffusometers.*

Apparatus.	Number of tubes.	Average length.	Total area of cross-section.	Ratio of length to diameter.
α	8	4·8166 cm.	0·7684 sq. cm.	13·9 approx.
β	4	2·0481 cm.	0·3044 sq. cm.	6·6 approx.
γ	1	4·014 cm.	1·768 sq. cm.	2·6 approx.

It may be mentioned that the measurements do not claim to be accurate to the last figures. The value of δ has been taken throughout as 0·45.

The volume of G_α=580 c.c. approx.; of g_α=80 c.c.; of G_β=660 c.c.; of g_β=260 c.c.; of G_γ=680 c.c.; of g_γ=360 c.c.

§ 5. *Results.*

Details of experiments are given in Table I. In the case of experiments α_1, α_2, α_3, β_1 and β_2 the first method of obtaining

TABLE I.

Experiment.	Interva's in hours.	Weight of flow in.gms.	Av.temp. during interval.	N.	n_0.	$_nK_N$ during interval.	$_nK_N$ average from start.
α_1	19·18	97·03	17·2°C.	0·2237	0·0004999	...	$1·639\times10^{-5}$
	28·32	96·02	17·2°C.	...	0·0004985	...	$1·655\times10^{-5}$
	24·52	97·36	17·2°C.	...	0·0004990	...	$1·641\times10^{-5}$
	23·83	95·37	17·2°C.	...	0·0005029	...	$1·643\times10^{-5}$
	23·75	94·77	17·2°C.	...	0·0005135	...	$1·642\times10^{-5}$
v. temp., 17·2	28·8	115·93	17·3°C.	...	0·0005125	...	$1·650\times10^{-5}$
α_2	74·33	63·13	17·6°C.	0·2238	0·002384	$1·668\times10^{-5}$	$1·668\times10^{-5}$
	99·97	74·70	17·7°C.	...	0·002699	$1·680\times10^{-5}$	$1·674\times10^{-5}$
	89·04	71·96	17·8°C.	...	0·002603	$1·735\times10^{-5}$	$1·693\times10^{-5}$
	71·53	56·54	17·8°C.	...	0·002504	$1·632\times10^{-5}$	$1·681\times10^{-5}$
	97·92	75·26	17·8°C.	...	0·002715	$1·724\times10^{-5}$	$1·690\times10^{-5}$
v. temp., 17·7	167·3	134·67	17·8°C.	...	0·002569	$1·706\times10^{-5}$	$1·693\times10^{-5}$
α_3	56·39	47·72	17·7°C.	0·2238	0·002397	...	$1·671\times10^{-5}$
	69·6	68·39	17·7°C.	...	0·002435	...	$1·837\times10^{-5}$
	93·69	73·18	17·8°C.	...	0·002580	...	$1·761\times10^{-5}$
	72·09	58·85	17·8°C.	...	0·002553	...	$1·760\times10^{-5}$
	96·36	77·42	17·9°C.	...	0·002545	...	$1·734\times10^{-5}$
	68·47	53·65	17·9°C.	...	0·002591	...	...
v. temp., 17·8	123·7	94·79	18·0°C.	...	0·002656	...	$1·714\times10^{-5}$
$_2\alpha_1$	71·33	30·8940	18·4°C.	0·2237	0·004785	$1·708\times10^{-5}$	...
	125·75	55·8370	17·9°C.	...	0·004396	1·675	...
	168·00	79·1620	17·7°C.	...	0·004175	1·621	...
v. temp., 17·8	166·00	78·5630	17·8°C.	...	0·004378	1·708	$1·672\times10^{-5}$
$_2\alpha_2$	144·7	57·4205	20·3°C.	0·0037	0·005737	$1·873\times10^{-5}$	...
v. temp., 20·4	382·5	144·6850	20·5°C.	...	0·005560	$1·734\times10^{-5}$	$1·772\times10^{-5}$
β_1	74·01	56·42	17·7°C.	0·2238	0·002609	...	$1·759\times10^{-5}$
	74·39	57·19	17·8°C.	...	0·002591	...	$1·760\times10^{-5}$
	98·9	75·10	18·0°C.	...	0·002511	....	$1·730\times10^{-5}$
	89·0	68·14	18·1°C.	...	0·002516	...	$1·723\times10^{-5}$
	71·5	53·54	18·1°C.	...	0·002527	...	$1·713\times10^{-5}$
	97·8	64·90	18·1°C.	...	0·002776	...	$1·697\times10^{-5}$
v. temp., 17·9	167·2	105·14	18·0°C.	...	0·002961	...	$1·685\times10^{-5}$
β_2	46·86	53·64	18·7°C.	0·2237	0·001714	$1·728\times10^{-5}$	$1·728\times10^{-5}$
	114·4	124·00	18·8°C.	...	0·001793	1·713	1·717
	506·2	52·34	18·9°C.	...	0·001865	1·698	1·712
	94·36	94·70	18·9°C.	...	0·001905	1·685	1·702
	71·44	68·97	18·9°C.	...	0·001977	1·682	1·700
	96·81	90·62	19·0°C.	...	0·002076	1·712	1·702
v. temp., 18·9	73·18	66·41	19·1°C.	...	0·002221	1·775	1·712
$_2\gamma_1$	93·4	40·2071	17·7°C.	0·2237	0·01191	$1·59\times10^{-5}$	...
	115·67	49·2115	17·7°C.	...	0·01210	1·59	...
	148·33	64·3615	17·4°C.	...	0·01162	1·56	...
v. temp., 17·5	239·67	100·7215	17·4°C.	...	0·01205	1·57	$1·58\times10^{-5}$
$_2\gamma_2$	117·33	74·2450	19·2°C.	0·2237	0·008740	$1·76\times10^{-5}$	...
	119·80	78·0071	18·6°C.	...	0·008319	1·65	...
	98·00	63·7300	18·3°C.	...	0·008545	1·70	...
v. temp., 18·7	92·30	60·1220	18·7°C.	...	0·008660	1·72	$1·71\times10^{-5}$
$_2\gamma_3$	96·33	47·3520	20·7°C.	0·2237	0·01251	$1·91\times10^{-5}$	...
	240·00	117·6237	20·7°C.	...	0·01195	1·82	...
	143·00	73·5120	20·7°C.	...	0·01169	1·86	...
v. temp., 20·8	144·20	74·2810	21·2°C.	...	0·01149	1·83	$1·85\times10^{-5}$

the flow was employed. In the case of β_2 the anti-vibrator was employed. In the case of experiments ${}_2\alpha_1$, ${}_2\alpha_2$, ${}_2\gamma_1$, ${}_2\gamma_2$ and ${}_2\gamma_3$ both the improved method of obtaining the flow and the anti-vibrator were employed. The experiments naturally fall into four groups, the results of which are summarised in Table II. The coefficient of diffusion at 18°C. (*i.e.*, k_{18}) is obtained with the aid of a formula kindly deduced for the authors by Mr. B. W. Clack—viz., $k_t = k_{18}[1+0{\cdot}023(t-18)]$. This equation is intended to apply to a 2·7 normal solution, which is little different from the 3N solution studied in this Paper. Mr. Clack informs the authors that the most probable value of k_{18} from his experiments (including his latest) is $1{\cdot}532\times10^{-5}$; thus the value of k_{18} ($1{\cdot}684\times10^{-5}$) given by the authors' analytical method is 10 per cent. greater than that given by Mr. Clack. The experimental conclusion of this Paper may be stated as follows: Assuming a linear relationship between the coefficient of diffusion and the temperature over the small range from 17·2°C. to 20·4°C., it is justifiable to co-ordinate the average of the temperatures and the average value of the coefficients; this gives that in the case of a solution containing 0·2237 grammes of potassium chloride to the cubic centimetre (a 3N solution) the "mean diffusivity" with respect to water is $1{\cdot}703\times10^{-5}$ (C.G.S. units) at a temperature of 18·5°C.

TABLE II.

Experiment.	Rate of flow in c.c. per hour.	Temp.	k.	k_{18}.	k_{18}.
α_1	4·020	17·2°C.	$1{\cdot}65\times10^{-5}$	$1{\cdot}681\times10^{-5}$	$1{\cdot}702\times10^{-5}$
α_2	0·794	17·7°C.	$1{\cdot}693\times10^{-5}$	$1{\cdot}703\times10^{-5}$	
α_3	0·816	17·8°C.	$1{\cdot}714\times10^{-5}$	$1{\cdot}722\times10^{-5}$	
${}_2\alpha_1$	0·463	17·8°C.	$1{\cdot}672\times10^{-5}$	$1{\cdot}680\times10^{-5}$	$1{\cdot}680\times10^{-5}$
${}_2\alpha_2$	0·382	20·4°C.	$1{\cdot}772\times10^{-5}$	$1{\cdot}680\times10^{-5}$	
β_1	0·931	17·9°C.	$1{\cdot}685\times10^{-5}$	$1{\cdot}689\times10^{-5}$	$1{\cdot}683\times10^{-5}$
β_2	1·003	18·9°C.	$1{\cdot}712\times10^{-5}$	$1{\cdot}677\times10^{-5}$	
${}_2\gamma_1$	0·428	17·5°C.	$1{\cdot}57\times10^{-5}$	$1{\cdot}588\times10^{-5}$	$1{\cdot}670\times10^{-5}$
${}_2\gamma_2$	0·641	18·7°C.	$1{\cdot}71\times10^{-5}$	$1{\cdot}684\times10^{-5}$	
${}_2\gamma_3$	0·498	20·8°C.	$1{\cdot}85\times10^{-5}$	$1{\cdot}738\times10^{-5}$	

§ 6. *Critical Consideration of the Results.*

Much of the work of this Paper was done in co-ordination with Mr. B. W. Clack; and apparatus β and apparatus γ were made because apparatus α gave results so different from his.

In all the calculations the value n_0 has been assumed to be that of the solution which comes from the diffusometer. In the case of the α and β experiments n_0 is so small that the assumption can produce no error of importance. The apparatus γ was made so as to include a diffusion tube which should be a close approximation to one used largely by Mr. Clack.* In the authors' experiments, however, involving as they do a continuous stream of water through the apparatus, the wide single tube gives less consistent results than the batteries of narrow tubes. It has been suggested to the authors that there may be an error due to a flow down certain tubes and up others. That this is extremely unlikely is proved by some earlier work† of one of the authors. Also the substantial agreement of the results given by the three pieces of apparatus, with various rates of flow, suggest that the flow has not a considerable influence. There is a possibility of a small error due to the reservoir G not being large enough to justify the assumption that the concentration at the bottom of the diffusion tubes is constant; but the results of Table I. certainly show no signs of a diminution in the calculated values of k which would result from a fall in the concentration. Taking the worst possible supposition—namely, that the solution in G continues homogenous throughout as it diminishes in concentration—it can be shown that the maximum error in the average value of k which could occur in an experiment lasting a month, in the case of the α and the β apparatus would be about 0·4 per cent., and in the case of the γ apparatus about 1 per cent. In practice, owing to the tendency of the lighter solutions to ascend to the top of the reservoirs, and owing to the time-lag in the diffusing power of a tube, the percentage errors will be much less than those calculated. Because perhaps of a variable amount being left in the compartment H the ratio between the quantity diffused and the time is not so constant as the corresponding ratio in Mr. Clack's experiments; nevertheless, the close agreement between the averages of α, β and γ prove that the results are not unworthy of consideration. And it would appear that there is almost certainly some, as yet unknown, factor or factors operating either in Mr. Clack's method, or in the analytical method described in this Paper, to which

* B. W. Clack, p. 43, "Proc." Phys. Soc., Vol. XXIV., Part I., December, 1911.

† A. Griffiths, p. 537, "Phil. Mag.," June, 1899. *It may be mentioned that the word "not" should appear before "appreciable" in the last line of the page.*

the consistent difference between the results obtained by the two methods must be attributed.

ABSTRACT.

This Paper represents an attempt to develop an analytical method of determining the coefficient of diffusion of a salt in water capable of giving consistent and accurate results.

The lower ends of a number of vertical and parallel diffusion tubes end in a reservoir of large capacity containing a solution of potassium chloride. The greater part of the reservoir is above the lower ends of the tubes, and by gravity the solution at the lower ends is kept at an approximately constant concentration. The upper ends of the tubes are covered with a cap provided with an outlet and an inlet tube. Water enters the cap by the inlet tube, and a weak solution containing the diffused salt leaves the cap by the outlet tube. Time, which may be as long as a fortnight, is allowed for the attainment of the steady state, and an individual experiment may last six weeks. The quantity diffused is obtained by chemical analysis. In the case of a solution containing 0·2237 gramme of potassium chloride to the cubic centimetre (a 3N solution) the "mean diffusivity" with respect to water is $1{\cdot}703 \times 10^{-5}$ (C.G.S. units) at a temperature of 18·5°C.

DISCUSSION.

Mr. B. W. Clack said that the difference of 10 per cent. between the authors' results and those obtained by him was not nearly so great as the discrepancies between the results of other workers on similar solutions. He showed slides containing the tabulated results of other observers for solutions of KCl and NaCl. These differed from one another by 20 to 30 per cent. He believed Dr. Griffiths had had some difficulty at first in the analysis of the dilute solutions. The method by which he had overcome this was, he thought, noteworthy.

Dr. S. W. J. Smith said he was interested in diffusion from the point of view of electrolytic phenomena. The way in which the diffusion depends on the concentration for different salts had an important bearing on the electrolytic behaviour of dilute solutions.

Dr. T. Barratt asked if variations of temperature might not account for the discrepancy between the authors' results and those of Mr. Clack. The temperature coefficient appeared to be about 7 per cent. for 3°C. Was it certain that in both methods the temperature was maintained constant to within this amount for the periods of six weeks or so required for an experiment? There was another effect of temperature variations. Any expansion of the solution in the reservoir had to take place through the diffusion tubes, and this would affect the results very seriously.

Dr. Griffiths, replying for the authors, said that the temperature had been kept constant to within 0·5°C. He had gone into all the possible errors due to temperature variations and they were not comparable with the 10 per cent. in question. His convective method gave results about 4 per cent. lower than the analytical method.

IX.—*An Apparatus for Evaluating Elliptic Integrals.* By A. F. Ravenshear.

Communicated by R. J. Sowter.

Received November 23, 1915.

In this apparatus, graphs of the three elliptic integrals can be described by mechanical means, and the value of the integral between any assigned limits found by measurement of the ordinates of the graph.

The principle of the apparatus is to use a hatchet scriber, which is controlled as regards direction, but is otherwise free to move anywhere over the surface of the paper. The hatchet cuts slightly into the paper, and as it moves makes a trace, which is the required graph. If the direction of the hatchet is kept constant, the trace is a straight line, but if the direction is subjected to a continuous change, the trace is a curve to which the hatchet is always tangent.

The graphs are made with reference to rectangular co-ordinates, the hatchet being directed so that the tangent of its inclination to the axis of X is always equal to the function of the abscissa (θ), under the sign of integration.

The integrals are dealt with in Legendre's form. Thus, taking the third integral as an example, we have to consider

$$\Pi(a, c, \theta) = \int \frac{d\theta}{(1+a \sin^2\theta)\sqrt{1-c^2 \sin^2\theta}}.$$

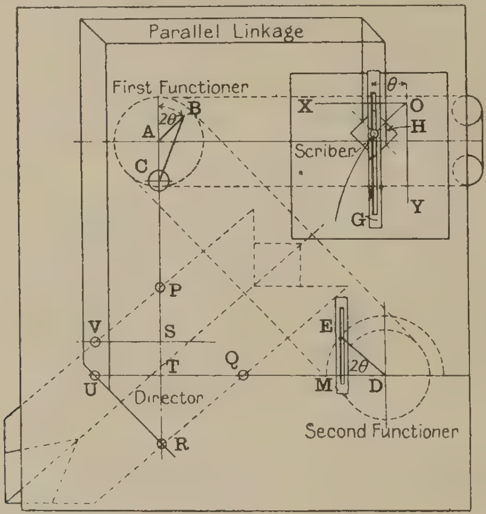
This is treated subject to the conditions that θ is limited to values from 0 to $\pi/2$, c to values from 0 to 1, and $(1+a \sin^2\theta)$ is always positive. These conditions cover all cases which can arise in dynamical and physical problems, and the resulting integrals are all finite.

I. The Chief Parts of the Apparatus.

The apparatus comprises five principal parts, supported partly above and partly below a rectangular board. In the right-hand top quarter is the hatchet scriber and its immediate accessories; in the left-hand top quarter the first functioner; in the right-hand bottom quarter the second functioner; in the left-hand bottom quarter a multiplying linkage and the director; and, lastly, extending from the director round the

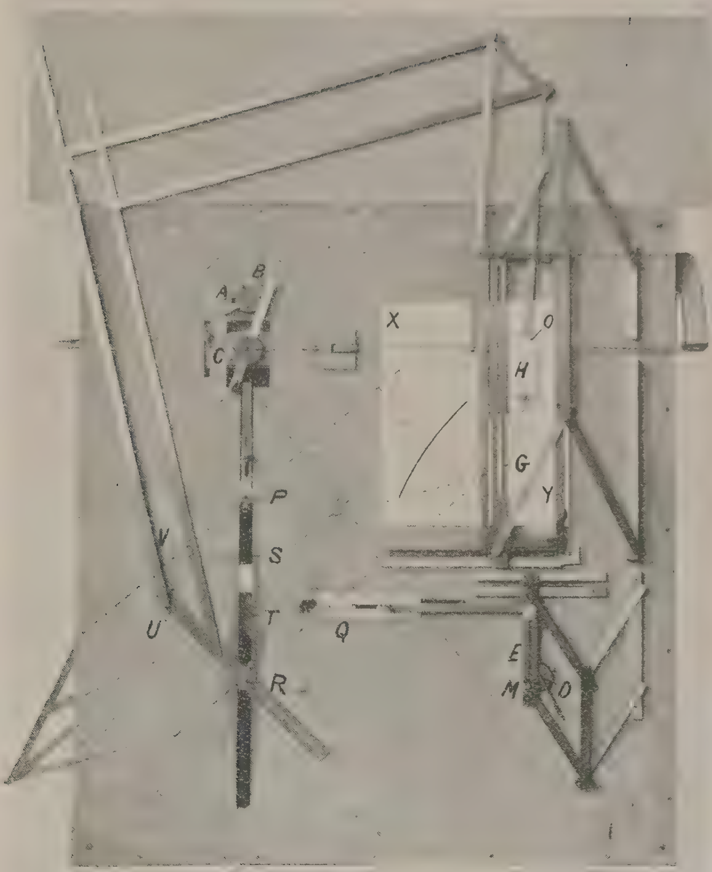
first functioner to the hatchet, a parallel linkage which controls the direction of the hatchet without otherwise restricting its movement.

1. *The Hatchet and its Immediate Accessories.*—The hatchet is on the underside of a plate H, capable of sliding freely along a guide G in the direction of the axis of Y, which guide can also move always parallel to itself in the direction of the axis of X. This guide is connected to a band passing over a pulley in the first functioner, so that a crank arm, AB, in the latter turns through two right angles, while the guide makes a full traverse of a length $\pi/2$.



2. *The First Functioner.*—In the first functioner, the crank arm, AB, mentioned above, forms one side of a variable triangle with a fixed base, AC, the length of the arm and the base being adjustable according to the value of the parameter c, so that the third side of the triangle, BC, always has the value $\sqrt{1-c^2 \sin^2\theta}$, where θ is half the angle swept out by the crank arm. By flexible transmission means, a slider, P, in the director is moved so that its distance from a certain fixed point, S, is always made equal to BC.

3. *The Second Functioner.*—In the second functioner another crank arm, DE, is made to turn at the same rate as the crank



To face p. 82.

arm, AB, in the first functioner. The length of this arm is adjustable according to the value of the second parameter, a, in the third integral, and from this arm is operated a second slider, Q, so that its distance from a fixed point T is always equal to $(1+a \sin^2\theta)$, where θ is half the angle swept out by the arm.

4. *The Multiplying Linkage and the Director.*—Under the board is a linkage which connects the first two sliders, P and Q, to a third slider, R, in such manner that the latter is moved when one or both of the first two sliders move so that its distance from the fixed point T is always equal to the product, $PS \times TQ$, of the distances of the other two sliders from their fixed points. On the top of the board is the director link, UR. This can be pivoted at either of the two fixed points, U or V, at unit distances respectively from T and S. When pivoted at V, the director link is guided by the first slider P, and when pivoted at U by the third slider, R, so that the tangent of its inclination to the axis of X is either

$$+\sqrt{1-c^2 \sin^2\theta}, \text{ or } -(1+a \sin^2\theta)\sqrt{1-c^2 \sin^2\theta}.$$

5. *The Parallel Linkage.*—This is a linkage which extends from the director link to the hatchet plate. Its attachment to the hatchet plate can be altered so that the hatchet can be placed either parallel to the director link or at right angles to it. The linkage maintains this relationship between the hatchet and the director link, but leaves the hatchet otherwise free. By these means the direction of the hatchet is controlled so that the tangent of its inclination to the axis of X is $\sqrt{1-c^2 \sin^2\theta}$ when the hatchet is kept parallel to VP, or $1/(1+a \sin^2\theta)\sqrt{1-c^2 \sin^2\theta}$, when the hatchet is kept at right angles to UR. For each point in the graph, taking the third integral as example, we therefore have

$$\frac{dy}{d\theta}=\frac{1}{(1+a \sin^2\theta)\sqrt{1-c^2 \sin^2\theta}},$$

whence

$$y=\int\frac{d\theta}{(1+a \sin^2\theta)\sqrt{1-c^2 \sin^2\theta}}.$$

II. The Manner of Using the Apparatus.

1. To draw the graph of the third integral for given values of the parameters, a and c, the crank arm, AB, in the first

functioner is set to a length $\frac{1}{2}-\frac{1}{2}\sqrt{1-c^2}$, and the block, C, through which passes the connecting-rod, BC, forming the third side of the variable triangle, is set so that its distance from the crank centre is $\frac{1}{2}+\frac{1}{2}\sqrt{1-c^2}$. The crank arm, DE, in the second functioner is set at $\frac{a}{2}$, the movable parts placed in the starting position, and the second slider, Q, is set at unit distance from the fixed point T. When this has been done, the first slider, P, is adjusted to unit distance from its fixed point, and the director link is pivoted at U and connected to the third slider, R. The parallel linkage is next connected with the hatchet plate so that the hatchet is at right angles to the director link. The hatchet is then pressed firmly on the card by the right hand, and the guide is caused to traverse by pulling with the left hand on a cord provided for the purpose, the hatchet plate being assisted in its movement in the direction of its length by the pressing hand.

2. The graph of the first integral is produced in exactly the same way as that of the third integral, after the crank arm, DE, has been set at zero, which has the effect of making the factor $(1+a\sin^2\theta)$ permanently unity.

3. To describe a graph of the second integral the director link is pivoted at V and connected to the first slider, P, and the hatchet is set parallel to the director link.

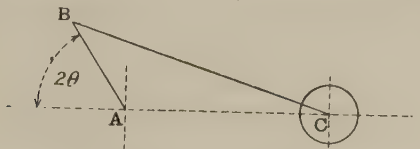
4. All the graphs of the second integral lie between the curve $y=\sin x$ and the straight line $y=x$, and the ordinates consequently never exceed $\frac{\pi}{2}$ in length. But the graphs of the first and third integrals are not confined within this region, and y in these cases may have large values, exceeding the limit fixed by the size of the apparatus. In such a case the graph may be completed by a simple device. When the hatchet plate reaches the upper end of the guide, the apparatus is stopped and held while the scriber is lifted slightly from the card and slid back along the guide towards the axis of X. This produces no effect upon its direction, and the graph can be continued from the new starting point merely displaced a distance equal to the displacement of the scriber.

III. The Geometry of the Mechanisms.

1. *The First Functioner.*—A crank arm, AB, turns about A, and the connecting-rod, BC, slides through a block which can

turn about C. The length of AB and of AC are both adjustable, and are given the following values :

$$AB=\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-c^2}\,;\ AC=\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-c^2}.$$

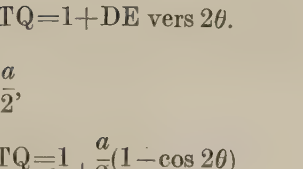


Hence, $$BC=\sqrt{AB^2+Ac^2+2AB\,.\,AC\cos 2\theta}$$

$$=\sqrt{(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-c^2})^2+(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-c^2})^2+2(\tfrac{1}{2}-\tfrac{1}{2}\sqrt{1-c^2})(\tfrac{1}{2}+\tfrac{1}{2}\sqrt{1-c^2})\cos 2\theta}$$

$$=\sqrt{1-c^2\sin^2\theta}.$$

2. *The Second Functioner.*—In this the pin on the crank arm, DE, engages with a movable guide, EM, which moves always parallel to the axis of Y, which guide is connected to the slider, Q. Since the slider, Q, is at starting at unit distance from T :



$$TQ=1+DE \text{ vers } 2\theta.$$

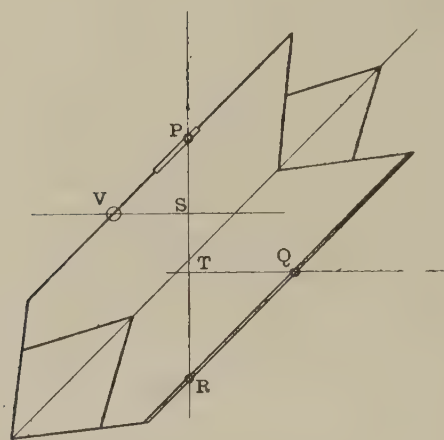
Since DE is made $\frac{a}{2}$,

$$TQ=1+\frac{a}{2}(1-\cos 2\theta)$$

$$=1+a\sin^2\theta.$$

3. *The Multiplying Linkage.*—In this, a pair of bars, VP and RQ, are connected by links in such manner that while they are free to approach and recede one from the other they are kept parallel. One of them is pivoted at the fixed point, V, and

they have sliding connections with the movable sliders, P, Q and R.



By similar triangles

$$\frac{\mathrm{TR}}{\mathrm{TQ}}=\frac{\mathrm{PS}}{\mathrm{VS}}.$$

But VS is unit length

$$\therefore\ \mathrm{TR}=\mathrm{PS}\,.\,\mathrm{TQ}.$$

ABSTRACT.

The apparatus produces by mechanical means a graph of the integral in rectangular co-ordinates. The graph is made by a hatchet scriber which is controlled as regards direction, but is otherwise free to move anywhere over the surface of the paper. Taking the third integral as an example.

$$\Pi(a, c, \theta)=\int\frac{1}{(1+a\sin^2\theta)\sqrt{1-c^2\sin^2\theta}}d\theta.$$

The apparatus comprises (1) the scribing appliance; (2) a mechanism which evaluates $\sqrt{1-c^2\sin^2\theta}$, where θ is the value of the abscissa of the middle point of the hatchet; (3) a mechanism which evaluates $(1+a\sin^2\theta)$; (4) a multiplying linkage which determines a length equal to the product $(1+a\sin^2\theta)\sqrt{1-c^2\sin^2\theta}$; and (5), a director which is variably inclined to the axis of X so that the tangent of its inclination is negative and equal to the above-mentioned product. By a parallel linkage, a line in the hatchet

plate at right angles to the hatchet is kept parallel to the director, so that the hatchet has an inclination φ to the axis of X such that

$$\tan \varphi = \frac{1}{(1+a \sin^2\theta)\sqrt{1-c^2\sin^2\theta}}.$$

Since the hatchet is always tangent to the graph which it describes, $dy/dx = \tan \varphi$, whence

$$y = \int \frac{1}{(1+a \sin^2\theta)\sqrt{1-c^2\sin^2\theta}} d\theta.$$

By suitable adjustments the apparatus deals in a similar way with the first and second integrals.

DISCUSSION.

Dr. A. Russell thought the instrument was very ingenious and likely to be of the greatest utility. The third elliptic integral was of frequent occurrence in electrical engineering and in hydrodynamics, and an apparatus for its easy evaluation would be welcomed by many.

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